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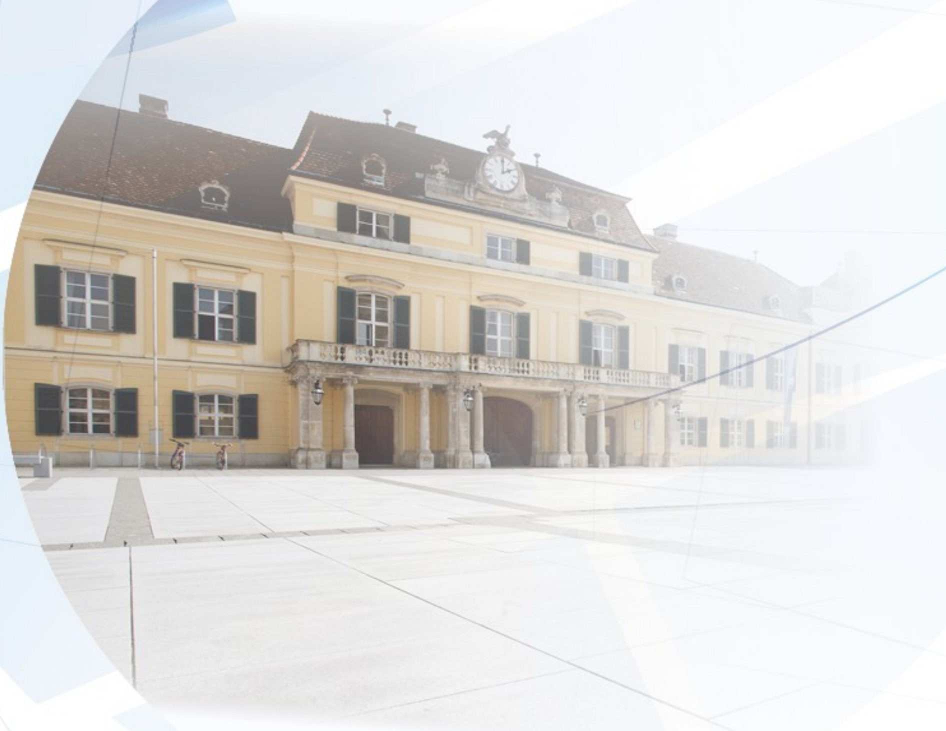

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Contents

Fairness and Equity via Concepts of Multi-Criteria Decision Analysis

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Abstract

We briefly discuss principles of fairness and equity in order to incorporate them in a mathematical method for the allocation of benefits or costs (the output) in a distribution problem, on the basis of the effort, the strength or the needs (the input) of the respective parties. Usually, input and output are multi-dimensional, and proportionality seems to be the leading principle. Therefore we employ several algorithmic ideas of Multi-Criteria Decision Analysis in order to support the solution of distribution problems, in particular the ideas underlying the Multiplicative AHP which was designed to process ratio information. We extend the method in order to cover the principles of progressivity, priority, and parity as well. Two examples, (a) the establishment of the member state contributions to the European Union, and (b) the allocation of seats in the European Parliament to the member states, show that the proposed method produces contributions and allocations with a higher degree of fairness and equity than the solutions adopted so far.

Key words

Fairness, equity, proportionality, progressivity, priority, parity, distribution criteria, desired ratios, logarithmic regression, geometric means.

1. Introduction

"All men agree that what is just in distribution must be according to merit in some sense, but they do not specify the same sort of merit" (Aristotle's Ethic, in the translation by J. Warrington, 1963). This statement briefly summarizes the two issues to be discussed in the present paper. First, the leading principle in fairness and equity is proportionality, which means that the benefits and the costs (the output) to be allocated to the parties in a distribution problem must be proportional to the effort, the strength, and/or the needs (the input) of the respective parties. Second, since input and output are usually measured under several criteria so that they are multi-dimensional, we have to weigh the distribution criteria in order to establish the aggregate quantities that must be proportional.

In this paper we present a general method to support a fair allocation of benefits and costs. As we will see (section 2), there are several principles of fairness. Proportionality is the leading one, not only in recent times but also in the Antiquity, see Aristotle (op. cit.) and the biblical parables of St. Matthew 25, 14 - 30 and St. Luke 19, 11 - 27. Many decisions, however, are based on the principles of progressivity, parity, or priority. According to the principle of progressivity, the output is increasingly assigned to the weaker or the stronger parties. Parity is the egalitarian principle whereby the output is divided into equal shares. The principle of priority awards the output in its entirety to one of the parties only. The choice of a particular principle is controlled by the assignment of specific values to certain parameters in the method. Since there is no clear border line between the above principles we design the method in such a way that the transitions from one principle to another one are as smooth as possible.

Since we are confronted with the subjective evaluation of multi-dimensional entities we turn to Multi-Criteria Decision Analysis (MCDA) for the calculation of fair allocations (section 3). By the principle of proportionality we are led to the algorithmic steps of the Multiplicative AHP because this method is particularly designed for the elicitation and the processing of ratio information. Thus, we use logarithmic regression in order to analyze desired-ratio matrices, and we apply geometric-mean calculations in order to work with a variety of distribution criteria. We will demonstrate that the method can easily be used under the other principles of fairness as well (section 4). All we have to do is to employ powers of the desired ratios. With exponents greater (smaller) than 1 we model the principle of progressivity (moderation), and in the limiting cases, when the exponents tend to infinity (zero), we proceed to the priority (parity) principle.

To illustrate matters, we first analyze the contributions of the member states to the European Union under the distribution criteria of population and Gross Domestic Product (section 5). With equal criterion weights we can reasonably enhance the fairness of the actual contributions. When we also take into account the national areas we obtain a really new, politically unexplored, set of contributions. This example highlights the typical features of our method. Similarly, we observe that the actual allocation of seats in the European Parliament deviates significantly from the allocation which is proportional to the size of the population. It can reasonably be approximated, however, by an allocation which is moderately proportional to the size of the population (section 6). Imposing certain additional constraints we can also prevent the total domination of the weaker countries by the stronger ones. We conclude this paper with a brief summary of open research questions (section 7).

2. Fairness and equity

Proportionality of input and output is by far the most widely discussed principle of fairness and equity. There are obvious utilitarian reasons for this. People are unwilling to make relatively high inputs unless they can look forward to relatively high outputs. Moreover, a person who can more effectively use a given scarce resource as a means of production should have a greater claim to its use. It is not unusual, however, that people moderate or amplify a proportional distribution. Young (1994) proposes four principles to determine a claimant's share: proportionality, progressivity, parity, and priority. The rationale for the progressivity principle, usually found in taxation schemes, is that those who are better off should pay at a higher rate because they can absorb the loss more easily. Under the egalitarian parity principle benefits and costs are allocated equally, even if the parties are unequal. The priority principle is an "all or nothing" principle which assigns absolute precedence to one party in the allocation of benefits and costs. It is usually applied to distribute indivisible goods.

Fishburn and Sarin (1994) discuss the issues of fairness and equity in the broad perspective of social choice. Fairness is based upon the preferences of individuals and groups, and upon the ways in which they perceive themselves in relation to others. Distributions that are fairest are those in which there is little or no envy among parties. Equity is based upon external ethical criteria, not on the specific preferences of individuals or groups. It is usually interpreted as some sort of equality, meaning that people are morally equal and should be treated with equal concern and respect. A distinction can also be made between fairness of outcome and procedural fairness (Linnerooth-Bayer et. al., 1994). In the first case the emphasis is on the results of the distribution process (do the parties agree with the proposed shares?), in the second case on the distribution process itself and on the role of the respective parties in it (did they receive a fair treatment, did they have a fair opportunity to explain their viewpoints?).

To conclude this section, we note that the literature on fairness and equity and on distributive justice is not only extensive but also confusing. Many highly similar concepts appear under different names. It is not always clear how they could be made operational (that is in fact the objective of the present paper). For more information we refer the reader to Deutsch (1975, 1985), Kasperson (1983), and Messick and Cook (1983).

3. A mathematical method for fair allocations

We consider a distribution problem with m criteria and n parties, first under the principle of proportionality. Let us take the symbol r_{ijk} to represent the desired ratio of the contributions c_j and c_k to be made by the respective parties under criterion i. Let us further introduce the symbol $R_i = \{r_{ijk}\}$ to stand for the matrix of the desired ratios under the i-th criterion. This matrix is positive and reciprocal, but not necessarily consistent, just like a pairwise-comparison matrix in the Analytic Hierarchy Process (AHP) of Saaty (1980). It may happen that $r_{ijk} \times r_{ikl} \neq r_{ijl}$ (for an example, see section 5). Let w_i stand for the weight assigned to the i-th distribution criterion. Following the mode of operation in the Multiplicative AHP of the first author (1993), we take the ratio c_j/c_k of any pair of contributions to approximate the desired ratios $r_{1jk},\ldots,r_{mjk}$ simultaneously, in the sense that we solve the contributions from the logarithmic-regression problem of minimizing

$$\sum_{i=1}^{m}\sum_{j<k} w_i\left\{\ln r_{ijk} - \ln c_j + \ln c_k\right\}^2 . \tag{1}$$

Actually, we carry out the unconstrained minimization by solving the associated linear system of normal equations with the variables $u_j = \ln c_j$, $j = 1,\ldots, n$. Obviously, the u_j have an additive degree of freedom. The c_j will accordingly have a multiplicative degree of freedom. A particular solution to the regression problem is given by

$$c_j = \prod_{i=1}^{m}\left(\sqrt[n]{\prod_{l=1}^{n} r_{ijl}}\right)^{w_i} , \tag{2}$$

which can be obtained if we calculate first the geometric row means of the matrices R_i and thereafter the geometric means of the row means. These operations may be interchanged without altering the final results. If the desired-ratio matrices happen to be consistent, the ratio of any pair of contributions is uniquely given by

$$\frac{c_j}{c_k} = \prod_{i=1}^{m}\left(\sqrt[n]{\prod_{l=1}^{n}\frac{r_{ijl}}{r_{ikl}}}\right)^{w_i} = \prod_{i=1}^{m}\left(r_{ijk}\right)^{w_i} . \tag{3}$$

Let us illustrate the above results via the allocation of fair contributions to the European Union, to be paid annually by the member states. If the respective contributions must be proportional to the size of the population and the Gross Domestic Product, we have two diverging requirements that can only approximately be satisfied. Suppose that equal weights are assigned to these distribution criteria. We take the ratio c_j/c_k of any pair of contributions to approximate the desired ratios

$$r_{1jk} = \frac{Pop_j}{Pop_k}$$

and

$$r_{2jk} = \frac{GDP_j}{GDP_k}$$

simultaneously by the solution of the above logarithmic regression problem. On the basis of formula (3) the ratio c_j/c_k can now be written as

$$\frac{c_j}{c_k} = \sqrt{\frac{Pop_j}{Pop_k} \times \frac{GDP_j}{GDP_k}}. \tag{4}$$

If we also want to the use the national area as a yardstick to set the contributions, we introduce the desired ratios

$$r_{3jk} = \frac{Area_j}{Area_k}.$$

By formula (3) the ratio of any pair of contributions is now given by

$$\frac{c_j}{c_k} = \sqrt[3]{\frac{Pop_j}{Pop_k} \times \frac{GDP_j}{GDP_k} \times \frac{Area_j}{Area_k}}, \tag{5}$$

at least if equal weights are assigned to the distribution criteria. The choice of the criterion weights in general is still under investigation. It is unclear, for instance, how large the weights should be in order to represent various gradations of relative importance of the distribution criteria. A detailed discussion of fair contributions to the European Union may be found in section 4.

A refinement of the model is to replace the r_{ijk} by powers $(r_{ijk})^{q_i}$. The positive exponent q_i introduces a moderation (amplification) of the desired ratios under the i-th distribution criterion if $q_i < 1$ ($q_i > 1$). For very small (very large) values of q_i there is a transition from the principle of proportionality to the principle of parity (priority). An application of the idea is presented in section 5, where we concern ourselves with the fair allocation of seats in the European Parliament.

4. Fair contributions to the European Union

The most recent data concerning the European Union may be found in the Eurostat Yearbook (1995). They reflect the situation until 1993. Table 1 shows the size of the population and the GDP of the respective member countries in that year. The resources of the European Union (63.75 billion ECU) consisted of customs revenues (16.8%), levies on agricultural imports and sugar storage (2.9%), a VAT-based levy (52.2%), a GDP-based contribution (25.2%), and non-attributable income (2.6%). The total contribution of

	Population 1993 in millions	GDP 1993 1000 million ECU	Population 1993 percentage of total	GDP 1993 percentage of total
Belgium	10.07	125	2.90	3.03
Denmark	5.18	85	1.49	2.06
Germany	80.98	1105	23.32	26.83
Greece	10.35	59	2.98	1.43
Spain	39.05	277	11.24	6.72
France	57.53	810	16.57	19.66
Ireland	3.56	37	1.03	0.90
Italy	56.96	658	16.40	15.97
Luxemburg	0.40	8	0.12	0.19
The Netherlands	15.24	205	4.39	4.98
Portugal	9.87	41	2.84	0.99
United Kingdom	58.10	709	16.73	17.21
Total	347.29	4119	100.00	100.00

Table 1. *Population and Gross Domestic Product of the 12 Member States of the European Union in 1993. Data from Eurostat Yearbook 1995, pages 72 and 196.*

	percentage of total	1000 million ECU	percentage of GDP	ECU per capita
Belgium	3.75	2.39	1.91	237
Denmark	1.80	1.21	1.42	233
Germany	29.07	19.03	1.72	235
Greece	1.34	1.01	1.72	98
Spain	8.29	5.19	1.88	133
France	18.18	11.56	1.43	201
Ireland	0.85	0.57	1.53	159
Italy	16.85	10.08	1.53	177
Luxemburg	0.21	0.17	2.10	420
The Netherlands	6.28	4.02	1.96	264
Portugal	1.63	0.91	2.21	92
United Kingdom	11.75	7.61	1.07	131
Total	100.00	63.75	1.55	184

Table 2. *Actual Contributions of the Member States to the European Union. Data from Eurostat Yearbook 1995, page 402.*

	percentage of total	1000 million ECU	percentage of GDP	ECU per capita
Belgium	2.90	1.84	1.47	184
Denmark	1.49	0.95	1.11	184
Germany	23.32	14.87	1.34	184
Greece	2.98	1.90	3.22	184
Spain	11.24	7.16	2.58	184
France	16.57	10.56	1.30	184
Ireland	1.03	0.66	1.78	184
Italy	16.40	10.45	1.59	184
Luxemburg	0.12	0.08	1.00	184
The Netherlands	4.39	2.80	1.37	184
Portugal	2.84	1.81	4.41	184
United Kingdom	16.73	10.67	1.50	184
Total	100.00	63.75	1.55	184

Table 3. *Possible Contributions to the European Union according to the Size of the Population.*

	percentage of total	1000 million ECU	percentage of GDP	ECU per capita
Belgium	3.03	1.93	1.55	192
Denmark	2.06	1.31	1.55	253
Germany	26.83	17.10	1.55	211
Greece	1.43	0.91	1.55	88
Spain	6.72	4.28	1.55	110
France	19.66	12.53	1.55	218
Ireland	0.90	0.57	1.55	160
Italy	15.97	10.18	1.55	179
Luxemburg	0.19	0.12	1.55	300
The Netherlands	4.98	3.17	1.55	208
Portugal	0.99	0.63	1.55	64
United Kingdom	17.21	10.97	1.55	189
Total	100.00	63.75	1.55	184

Table 4. *Possible Contributions to the European Union according to the Gross Domestic Product.*

	percentage of total	1000 million ECU	percentage of GDP	ECU per capita
Belgium	2.99	1.91	1.53	190
Denmark	1.77	1.13	1.32	218
Germany	25.22	16.07	1.45	198
Greece	2.08	1.32	2.24	128
Spain	8.76	5.58	2.01	143
France	18.21	11.60	1.43	202
Ireland	0.97	0.62	1.68	174
Italy	16.32	10.40	1.58	183
Luxemburg	0.15	0.10	1.25	250
The Netherlands	4.72	3.01	1.47	198
Portugal	1.69	1.08	2.63	109
United Kingdom	17.11	10.91	1.54	188
Total	100.00	63.75	1.55	184

Table 5. *Possible Contributions to the European Union according to the Geometric Mean of the Population and the Gross Domestic Product.*

	National Area 1993 in 1000 sq km	National Area 1993 percentage of total	Population Density inhabitants per sq km
Belgium	31	1.3	330
Denmark	43	1.8	120
Germany	357	15.1	227
Greece	132	5.6	78
Spain	505	21.3	77
France	549	23.2	105
Ireland	70	3.0	51
Italy	301	12.7	189
Luxemburg	3	0.13	152
The Netherlands	41	1.7	370
Portugal	92	3.9	107
United Kingdom	244	10.3	238
Total	2368	100.0	147

Table 6. *National Area and Population Density of the Member States of the European Union. Data from Eurostat Yearbook 1995, page 168.*

	percentage of total	1000 million ECU	percentage of GDP	ECU per capita
Belgium	2.35	1.50	1.20	149
Denmark	1.85	1.18	1.39	228
Germany	22.07	14.07	1.27	174
Greece	3.01	1.92	3.25	186
Spain	12.24	7.80	2.82	200
France	20.48	13.06	1.61	227
Ireland	1.47	0.94	2.54	264
Italy	15.59	9.94	1.51	175
Luxemburg	0.15	0.10	1.25	250
The Netherlands	3.49	2.22	1.08	146
Portugal	2.32	1.48	3.61	150
United Kingdom	15.00	9.56	1.35	165
Total	100.00	63.75	1.55	184

Table 7. *Possible Contributions to the European Union according to the Geometric Mean of the Population, the Gross Domestic Product, and the National Area.*

	Pop	$Pop^{0.9}$	$Pop^{0.8}$	$Pop^{0.7}$	Actual
Belgium	16	19	22	25	25
Denmark	8	10	13	16	16
France	94	91	88	85	87
Germany	132	124	116	108	99
Greece	17	20	22	26	25
Ireland	6	7	10	12	15
Italy	93	91	88	84	87
Luxemburg	1	1	2	3	6
The Netherlands	25	28	30	33	31
Portugal	16	19	22	25	25
Spain	64	65	65	65	64
United Kingdom	95	92	89	85	87
Total	567	567	567	567	567

Table 8. *Number of seats in the European Parliament, proportional to the size of the population of the member states (column* 2)*, moderately proportional to the size of the population (columns* 3 - 5)*, as well as the actual allocation of seats in* 1993 *(column* 6)*. Data from Eurostat Yearbook* 1995.

each of the member countries in 1993 is exhibited in Table 2, which also shows how the contributions are related to the national economies. The contributions expressed as a percentage of the GDP vary between 1.07 (United Kingdom) and 2.21 (Portugal), the contributions in ECU per capita between 92 (Portugal) and 420 (Luxemburg). Since the ratio of the smallest to the largest contribution per capita is roughly 1:5, fairness is far to seek.

The Tables 3 and 4 show that contributions proportional to the size of the population or the GDP only (one single distribution criterion) are also unfair. When only the size of the population is used, there is indeed a uniform contribution of 184 ECU per capita, but the contributions expressed as a percentage of the GDP vary between 1.00 (Luxemburg) and 4.41 (Portugal). Similarly, with the GDP as the unique yardstick, the contributions are uniformly set to 1.55% of the GDP, but the contributions in ECU per capita vary between 64 (Portugal) and 300 (Luxemburg). So, the ratio of the smallest to the largest contribution per capita is roughly 1:5, although the GDP is generally considered to be a proper yardstick for a country's ability to pay (see also Beckermann (1980)).

A considerable improvement is obtained when we allocate the contributions on the basis of the size of the population and the GDP simultaneously, with equal weights so that formula (4) applies. Table 5 shows that the contributions expressed as a percentage of the GDP now vary between 1.25 (Luxemburg) and 2.63 (Portugal), whereas the contributions in ECU per capita vary between 109 (Portugal) and 250 (Luxemburg). Under both distribution criteria the ratio of the smallest to the largest contribution is reduced to 1:2.

The third distribution criterion that may come up in the discussions on fairness and equity is the national area. A large area has many possible advantages for a country: a large amount of arable land and fresh water to support agriculture, large mountainous regions to support tourism and water winning, large spaces to enhance the quality of life, and/or large mineral deposits or fossil-fuel supplies. The national areas and the population densities of the member states may be found in Table 6. Table 7 shows the possible contributions to the European Union when the size of the population, the GDP, and the national area are used with equal weights to distribute the total burden. Formula (5) is clearly applicable. The contributions now vary between 1.08% (The Netherlands) and 3.61% (Portugal) of the GDP, and between 146 ECU (The Netherlands) and 264 ECU (Ireland) per capita. The ratio of the smallest to the largest percentage of the GDP is higher than 1:2 now, but this may have a good reason: the population densities in the European Union vary widely, between 51 inhabitants per km^2 (Ireland) and 370 (The Netherlands). The third distribution criterion is clearly an incentive for the member states to exploit their natural resources more effectively.

Note. In the present paper we ignored the attempts of various European countries to use the principle of "juste retour" in order to regain their contributions to the Union as much as possible.

5. Fair seat allocations in the European Parliament

With some moderations, the principle of proportionality can also be used to "explain" the seat allocation in the European Parliament as a function of the size of the population of the member states. Table 8 shows the respective seat allocations (calculated with Webster's apportionment rule, see Young (1994)) when the ratio s_j/s_k of any pair of national delegations is taken to be

$$\frac{s_j}{s_k} = \left(\frac{Pop_j}{Pop_k}\right)^q, \; q = 1, 0.9, 0.8, 0.7.$$

With $q = 1$, we have pure proportionality on the basis of the size of the population. Smaller values of q lead to more equality between the delegations. Remarkably enough, the seat allocation for $q = 0.7$ practically coincides with the actual seat allocation in the European Parliament. At the extreme ends of the spectre, however, we find some discrepancies. There are two countries with an over-representation, Ireland and Luxemburg, and one with an under-representation, Germany. An issue to be put on the political agenda?

Let us now use the seat-allocation problem in order to illustrate the introduction of certain constraints in a distribution problem. Barzilai and Lootsma (1997), considering the power relations in groups, proposed to limit the ratio of any two contributions to the range between $^1/_{16}$ and 16 in order to prevent the total domination of the stronger parties over the weaker ones. Table 8 shows in column 6 that these constraints are actually satisfied in the European Parliament despite the discrepancies between Luxemburg and Germany. There are at least two ways to approximate or to achieve this via our method as well.

The first approach does not exactly attain the goal. We truncate the desired ratios. Setting

$$\alpha_{jk} = \frac{Pop_j}{Pop_k},$$

we define the desired-ratio matrix R with elements r_{jk} such that

$$r_{jk} = \begin{cases} 1/16, & \alpha_{jk} < 1/16, \\ 16, & \alpha_{jk} > 16, \\ \alpha_{jk}, & 1/16 \le \alpha_{jk} \le 16. \end{cases}$$

This matrix is inconsistent, but it can easily be analyzed via logarithmic regression (see section 3): the geometric row means provide the number of seats per member state, in general as non-integer numbers with a multiplicative degree of freedom.

The second option is to minimize the logarithmic-regression function (1) under the constraints

$$- \ln 16 \leq \ln c_j - \ln c_k \leq \ln 16,$$

for any j and k. This is a convex quadratic-programming problem when we take the logarithms of the contributions to stand for the variables. Since there are only differences of logarithms in the problem formulation, any solution has an additive degree of freedom. Thus, the number of seats per member state is produced in the form of non-integer numbers with a multiplicative degree of freedom. One needs an apportionment rule to obtain a workable seat allocation.

6. Epilogue

We conclude this paper with a brief sketch of some open research questions and some possible applications of our proposal to model the principles of proportionality, progressivity, parity, and priority via desired-ratio matrices and logarithmic regression.

Our method is clearly applicable only as soon as a major decision has been made: what are the relevant distribution criteria? This question is beyond the scope of the method. It is one of our objectives, however, to model the relative importance of the criteria (see Lootsma (1996)) and to study the parameters controlling the convergence towards the principle of parity (moderation) or priority (amplification).

It sometimes happens that additional constraints are imposed on the possible allocations (Ramanathan and Lootsma (1996)). Each party should have a minimum contribution, for instance, and certain coalitions of parties should not be permitted to dominate the scene. This may lead to a problem formulation where we are confronted with constrained minimization of the logarithmic-regression function (1). In general, as soon as we have a problem which is formulated in terms of the contributions and also in terms of their logarithms, we may be running up against a non-convex problem with local, non-global solutions.

Just like in MCDA, we plan to extend the method so that it can be used in group-decision making. The method should not only identify the opinions of the individual members of the group. It should also come up with a possible compromise solution which takes into account the relative power of the members.

A possible application is the allocation of hazardous waste to certain regions with widely varying levels of urbanization, industrialization, affluence, natural beauty, unemployment, and rural development. The potential of the method is still to be explored here. It can also be applied on any world issue where consensus has to be obtained after negotiations by all countries involved in a distribution problem. This is a typical problem that is regularly encountered in the global negotiations on actions to deal with the environment and with climatic change. We could possibly identify the important criteria and the preferences of the individual countries, and we could analyze the power structure of the negotiating parties, in order to arrive at a fair compromise solution.

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