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Measuring in Advance the Accuracy of Population Forecasts

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Foreword

Some compilations are now available in which forecasts are compared with the population that eventuated; these are properly called *ex post*, after the event. They are useful for showing the errors to which the *process* of projection is subject in general; inapplicable to the individual forecast at the time it is made, they can say how well a particular forecaster has done on the average of a number of attempts, or how easy or difficult it has been to forecast for a particular country. This kind of checking has to wait 10, 20, or more years after the forecasts were made.

What is needed is an *ex ante* method, i.e. one that can be published at the same time as the forecast. The assumptions on which the *ex ante* method of this paper is based are that the uncertainty of forecast is determined by the fluctuations that will occur, and that the amount of fluctuation in the future will be the same as in the past. These assumptions are applied separately to fertility rates, to mortality improvement, and to net immigration, the future population being a simple function of these.

Such a set of assumptions needs some kind of confirmation or calibration; this was done with the known amount of average *ex post* error to which population projections made by the UN and other agencies have been subject over the past 30 years. It was found that selecting a random past year, assuming its fertility will hold for a future year, and replicating, gave on the average about the same amount of error *ex ante* as the known *ex post*.

One would have thought something quite different: that taking a random year from the past and applying it to the future is a forecast far inferior to the careful work done by official agencies with their trained staffs, and so would show too much variation; that is not what was found. Of course this result for Canada will have to be checked for other countries.

In one respect this should not surprise. We should know that forecasts track the changing fertility. The United Nations gave the medium variant Canadian 2020 population as 34,226,000 in its 1980 assessment; 33,621,000 in the 1982 assessment; 32,525,000 in the 1984 assessment; and 31,587,000 in the 1986 assessment. Those making the calculation successively modified downwards their medium variant projection. The difference between the 2020 value as shown in 1980 and in 1986 is more than a whole standard deviation as calculated in this paper.

Let us not deduce from this that there is some better way to proceed. We might try extrapolating the past trend in births, mortality improvement and migration, but extrapolation in fact gives a worse estimate of future input than taking random items from the past. For instance extrapolation from the last 30 years would give zero births within the first quarter of the 21st century.

The method as applied to Canada shows that the error for 40 years in the future may be so large—only an 85 per cent probability that the total population will be between 30 and 35 million—that the estimate could well be judged useless. On the other hand the corresponding range for 20 years in the future is reasonably narrow at 28 to 30 million.

No claim is made for the absolute amounts in the population projections of this paper, which is exclusively concerned with establishing uncertainty, and not with levels.

Nathan Keyfitz
Leader, Population Program

Abstract

Supposing only that future variability in fertility, mortality improvement, and migration will be the same as past variability, and that it is the variability that creates the uncertainty in population forecasts, permits an *ex ante* estimate of uncertainty. This is calculated by taking the fertility level of a random past year, the mortality improvement of a random past five-year period, and the net immigration of a random past year. The future population that is shown by 1000 such random choices of each of the three input variables for each projection cycle gives the variability to which individual estimates are subject.

We need to check this *ex ante* calculation with what actually happens, and are able to do so for past projections, comparing a large number of medium projections made by the United Nations and other bodies with what subsequently occurred. It turned out that this *ex post* estimate was slightly **higher** than the *ex ante* as shown by the simulation.

Thus, contrary to appearances, the method of this paper does not exaggerate the variability but if anything underestimates it.

Estimating variability is less important for the total than it is for certain ratios. Of these the most sought after is that of old people to the number who are at working age, a ratio that determines the tax rate needed to provide old age pensions. The uncertainty of this and any other function of future population can be found by the same simulation.

Acknowledgement

The author is grateful to the U.S. National Science Foundation for financial support of the calculations here described.

MEASURING IN ADVANCE THE ACCURACY OF POPULATION FORECASTS

Nathan Keyfitz

Three assumptions suffice for establishing the precision of our knowledge of future population:

1) The uncertainty of future population depends on the uncertainty in the inputs: births, mortality improvement, and migration.

2) The uncertainty in any variable depends on its variability.

3) There is no reason to suppose that future variability will be either greater or less than past variability.

1) The first of these assumptions follows from the basic accounting identity. If fertility, mortality and immigration are the inputs to the population process, of which the output is the number of people, the model by which input passes to output is simply: population at time t equals population at time $t-1$ plus births minus deaths plus net migration, all in period $(t-1,t)$. In symbols, if p is population, b is the numbers of births in any period, say n years, d the number of deaths, m the number of migrants, then Δp, the change in population, is equal to:

$$\Delta p = b - d + m ,$$

so that with independence the variance in the population change over the next n years is

$$\mathrm{VAR}(\Delta p) = \mathrm{VAR}(b) + \mathrm{VAR}(d) + \mathrm{VAR}(m) .$$

2) Uncertainty and variability are either the same thing or intimately related. Any variable that does not change can be perfectly estimated for the future from its present value. One that changes twice as much as another would seem to have future values twice as uncertain.

3) It cannot convincingly be argued that future fertility variation will be less than past, though it has been casually asserted that we cannot have another baby boom. Many of the past errors in population forecasts arose from supposing that the future births, etc., would be close to their values at the jumping off point. We cannot tell in what direction the change will be, but that there will be change is certain.

We may well be in a better position to estimate the error of an estimate of future population than to estimate the future population itself. Given this, and the importance of knowing errors in advance, it is surprising that estimates of error are so rarely published as accompaniments of the estimate of the population.

If future birth, death, and migration rates will vary about in the same degree as in the past, this symmetry between past and future will enable us to estimate the uncertainty of population forecasts *ex ante*, i.e. at the same time as we make the estimates themselves. *Ex post* estimates (Keyfitz 1981, Stoto 1983) are obtainable only long after, by comparing the forecast with the population that materialized. Whatever the method for calculating *ex ante* estimates we can ultimately check it with *ex post*, and that will be done in this paper.

Since to construct the *ex ante* measure we have to consider separately the three components of population change, we will have an opportunity to estimate the contribution of each component to the uncertainty. We start with fertility, then go on to mortality improvement and migration. For each we will take the mean from the past without claiming anything for this as an estimate of the future. Suggestions for improving forecasts are made elsewhere; here we show a way to assess their error.

Ex ante methods based on models are found in Spencer and Alho (1985) and Tulipurkar (1989). In contrast to these, the present paper uses simple simulation, which is to say repeats the projection, typically 1000 times.

FERTILITY

Developed countries now show a cross sectional net reproduction rate about 0.8, down from over 1.5 30 years ago. Projecting the trend of the past 30 years would bring us to zero births in the next generation. Thus, extrapolating trends has little place in population projection, and supposing that past levels continue into the future is much less objectionable.

In accord with its aim of contributing not to projection itself, but only to our knowledge of the uncertainty of projections, this paper asks the question in relation to fertility: how uncertain is whatever value is to be used for the forecast? Whether or not it is correct or acceptable to take the expected future fertility the same as the past, for estimating uncertainty it is reasonable to suppose that future variation will be about the same as past variation. On this principle we choose a past year at random, and taking its rate as the fertility for a future five-year period, carry out the projection, and then repeat 999 times the random selection from the past and the projection.

MORTALITY

For mortality the story is different in that there is a clear time trend in the rates, but with random-seeming year-to-year differences. We know that mortality will continue to improve—unless catastrophe occurs, and that we omit from the calculation. What we do not know is how fast the improvement will be; will it be at the rapid pace shown in the late 1970s, or as slow as in the 1920s? Here our procedure is to choose at random a past five year interval, and suppose that the improvement will take place at the pace shown in this interval. Once chosen we will suppose that that is the rate of improvement that will continue into the future.

One way to do a probability distribution is to take the two outside possibilities, and for any time assign a random positive (strictly non-negative) weight to each, with the two weights adding to unity. In short one would choose at random a value intermediate between the highest and the lowest considered possible, with equal probability for any point on the interval, which is to say on a uniform distribution.

Thus for mortality the two outside possibilities might be the improvement of the poorest five years and the best five years, projected by geometric progression on the q_x, then converted to L_x. Following is the life expectancy $(\overset{o}{e}_0)$ that results:

High survivorship projects the ratio of q_x for 1981 life table to that of 1971
Low survivorship projects the ratio of q_x for 1931 life table to that of 1921

	1981	1986	1991	1996	2001	2006	2011	2016	2021	2026
High	75.517	76.546	77.481	78.334	79.119	79.842	80.512	81.133	81.712	82.250
Low	75.517	75.663	75.798	75.923	76.040	76.148	76.248	76.341	76.428	76.509

and the resultant projections (in thousands), with average fertility and migration, are

	1980	1985	1990	1995	2000	2005	2010	2015	2020
High	24,089	25,579	27,061	28,354	29,494	30,478	31,407	32,340	33,232
Low	24,089	25,534	26,925	28,090	29,068	29,857	30,563	31,242	31,842

and the ratio of the number of persons 65 and over to the number 20 to 64 is

	1980	1985	1990	1995	2000	2005	2010	2015	2020
High	0.166	0.178	0.195	0.206	0.215	0.222	0.241	0.281	0.327
Low	0.166	0.176	0.190	0.196	0.199	0.200	0.213	0.243	0.278

As is to be expected, high survivorship gives the higher ratio of old people to working people, higher by nearly 20% on the mortality of the 1970s than on the mortality of the 1920s.

In this paper I have made a probability distribution for future mortality, by selecting a random past interval of five years, and apply the improvement of that particular five-year period to the future. We know that mortality will improve, barring a catastrophe that there is no way of taking into account, and the only guide that is apparently available is past improvement. And since we have no idea of what past improvement will be found in the future, the most appropriate thing to do is to select a past period at random. We do not say that the mortality of this past period is what will occur in the future, only that from where we are now the chance of it is as good as the chance of any other past period.

Table 1. Future life expectancy and future population, when mortality improvement is selected from five-year intervals in the past, and fertility and migration are held fixed.

Future values of e_0 with continuation of life table

improvement between	1986	1991	1996	2001	2006	2011	2016	2021	2026
1921 and 1926	76.28	76.98	77.64	78.26	78.83	79.37	79.88	80.35	80.79
1926 and 1931	75.02	74.50	73.95	73.37	72.76	72.12	71.46	70.76	70.04
1931 and 1936	75.87	76.17	76.43	76.65	76.83	76.98	77.10	77.20	77.26
1936 and 1941	76.12	76.63	77.07	77.45	77.78	78.07	78.32	78.54	78.73
1941 and 1946	76.46	77.30	78.06	78.74	79.37	79.95	80.48	80.97	81.43
1946 and 1951	76.36	77.07	77.69	78.22	78.69	79.12	79.50	79.84	80.16
1951 and 1956	76.42	77.22	77.93	78.58	79.16	79.69	80.17	80.62	81.03
1956 and 1961	76.23	76.90	77.53	78.12	78.68	79.21	79.70	80.17	80.62
1961 and 1966	75.85	76.15	76.42	76.66	76.89	77.09	77.26	77.42	77.56
1966 and 1971	76.09	76.60	77.06	77.48	77.86	78.21	78.51	78.79	79.03
1971 and 1976	76.28	76.96	77.59	78.17	78.70	79.20	79.68	80.12	80.54
1976 and 1981	76.81	77.96	79.00	79.94	80.79	81.56	82.26	82.89	83.48

Future values of population total (thousands of persons) with continuation of life table

improvement between	1980	1985	1990	1995	2000	2005	2010	2015	2020
1921 and 1926	24089	25569	27031	28297	29406	30355	31246	32138	32980
1926 and 1931	24089	25496	26808	27850	28659	29227	29658	30001	30203
1931 and 1936	24089	25540	26944	28126	29128	29946	30680	31383	31999
1936 and 1941	24089	25551	26978	28193	29232	30096	30889	31663	32362
1941 and 1946	24089	25575	27048	28330	29454	30418	31318	32215	33062
1946 and 1951	24089	25566	27020	28275	29364	30282	31134	31981	32772
1951 and 1956	24089	25569	27032	28300	29410	30359	31247	32130	32957
1956 and 1961	24089	25567	27023	28281	29371	30299	31161	32015	32815
1961 and 1966	24089	25545	26957	28148	29156	29975	30711	31431	32088
1966 and 1971	24089	25558	26995	28223	29275	30153	30958	31747	32474
1971 and 1976	24089	25565	27017	28269	29354	30272	31125	31976	32778
1976 and 1981	24089	25593	27101	28433	29619	30657	31644	32639	33597

It is possible to improve on this by giving a larger probability of choice to the more recent time. Perhaps the probability could be made proportional to the time from the start of the record, that has not been applied here. For this, if there are n periods in the record, then the probability assigned to the ith period would be $2i/\{n(n+1)\}$. If that gives too much weight to the most recent time then one could modify it by having a fixed probability, a/n, where $0 < a < 1$, plus an increasing one, $2i(1-a)/\{n(n+1\}$. Then the probability for the ith unit would be

$$a/n + 2i(1-a)/\{n(n+1)\} \quad .$$

If there is a strong overall trend in the past history, then the choice of a is of some importance. If there is no clear trend then setting $a = 1$ is best. Our ignorance in respect of future mortality can be expressed as uncertainty on what past time describes the pace of improvement for the future.

It could well indeed argued that the recent past is more likely to be reproduced than the distant past, so we should weight the selection. Since the regression over time is nearly horizontal this is a matter of principle rather than of computational importance—it will change the result little. I have used in the expression above with $a = 1$; others might prefer a somewhat less than 1. The arbitariness of a is irremovable.

IMMIGRATION

For net immigration we again chose at random from the past record, in each projection cycle. Migration turns out to be a small part of the uncertainty on total population, and an even smaller part of the future ratios of old people to working people.

We need the net immigration by age for the projection, and that seems unrelated to the existing population. We could of course take age-specific immigration rates by age with the denominator consisting of the population initially present, which would require us to think of the population as exposed to the risk of immigration, to use the language of the life table. But it seems better for the present purpose to accept a figure of net immigration, unrelated in its variation to the variation in the population as a whole. That number would be derived from the past as well; we would suppose for the future distribution the same as the past has shown, but now only take the absolute total, and for each future value the distribution by age given by the most recent data.

OUTCOMES: TOTAL POPULATION OF CANADA

Here is a sample of the results (for total population of Canada, in thousands) consisting of two random projections for the years 1980–2020 with all three inputs taken at random from the record for 1921–1981:

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Trial 1	24089	25664	27032	28364	29525	30571	31556	32456	33256
Trial 2	24089	25597	26897	28159	29245	30212	31108	31914	32623

and here are 20 totals for the year 2020:

33256	31792	31968	32957	30463	33571	32773	32634	32518	30844
32623	36093	29794	34143	34131	32858	31580	32850	34974	30887

For the mean of the 20 drawings we have

	1980	1985	1990	1995	2000	2005	2010	2015	2020
mean of 20	24089	25640	26962	28217	29275	30212	31102	31921	32635

and for their standard deviation:

	1980	1985	1990	1995	2000	2005	2010	2015	2020
SD of 20 trials	0	111	234	368	518	704	939	1219	1535

We go on to a larger simulation of 1000 trials:

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean of totals	24089	25590	26858	28054	29048	29922	30757	31525	32190
SD of totals	0	168	345	525	712	930	1197	1510	1849

and from this sample we read a 2020 population of 32,190,000, and with odds of 2:1 what will materialize will fall in the range 32,190,000 ± 1,849,000, or roughly 30,000,000 to 34,000,000. That will in fact turn out to be somewhat of an underestimate of the range in relation to our *ex post* survey of actual successes and failures.

And here is another sample of 1000; with samples so large it would be surprising if there turned out to be much difference from the first:

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean of totals	24089	25596	26868	28066	29062	29938	30774	31544	32211
SD of totals	0	163	336	512	697	913	1181	1496	1839

We can provide information on the process in other ways—following is the projected population to 2020 for the first trial and every 45^{th} subsequent trial when a set of 990 outcomes is arranged in numerical order:

25634	29235	29922	30437	30741	31036	31263	31427	31578	31778	31969
32164	32423	32610	32821	33061	33305	33533	33774	34121	34442	35214

Thus for 2020 one item out of the thousand falls as low as 25,634,000, and 44 are above 35,214,000; the highest is 38,382,000. That is to say with probability 0.998 the past record bounds the 2020 population only between 25,634,000 and 38,382,000. This is an outrageously broad range, but as will be seen later, it is consistent with what the *ex post* calculation shows to be the error actually occurring.

OUTCOMES: RATIO OF PERSONS 65 AND OVER TO THOSE 20 TO 64

We are also interested in the ratio of those 65 and over to those 20–64. Because of its relation to social security this ratio is especially important to know. The means and standard deviations are given below for the ratio as obtained in one set of 1000 trials:

Mean and standard deviation (x 1000) for ratio of 65+ to 20-64

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean of ratios	166	176	184	194	199	202	217	252	292
SD of ratios	0	1	3	5	8	11	14	19	26

The standard deviations are higher proportions for the ratios than for the totals, for 2020 0.026/0.292 or about 9 per cent against 6 per cent for the total. This is because the numerator and denominator of the ratio vary in some degree independently, being impacted differently by fertility, mortality, and migration.

And a second sample of 1000 for the ratio, that comes out virtually identical with the first:

	Mean and standard deviation (x 1000) for ratio of 65+ to 20-64								
	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean of ratios	166	177	184	195	199	202	217	252	292
SD of ratios	0	1	3	5	8	11	15	20	27

DISTRIBUTIONS

For the purpose of using our simulations to make statements on probabilities, it is convenient to know not only the standard deviation but the full distribution of outcomes. That turns out to be normal, at least for the totals. Table 2 gives the distribution and the fitted normal curve.

Table 2. Distribution of the simulated Canadian population for 2020 when birth, mortality improvement, and migration are selected at random from past experience, along with the fitted normal curve.

Interval	Simulated	Fitted Normal
-28,000,000	11	10
28,000,000-	6	10
28,500,000-	25	18
29,000,000-	27	29
29,500,000-	48	43
30,000,000-	51	60
30,500,000-	68	77
31,000,000-	86	93
31,500,000-	116	104
32,000,000-	100	108
32,500,000-	128	104
33,000,000-	99	94
33,500,000-	63	78
34,000,000-	58	60
34,500,000-	37	44
35,000,000-	39	29
35,500,000-	17	18
36,000,000-	11	10
36,500,000-	9	6
37,000,000-	1	3
37,500,000-	1	2

The observed distribution does not significantly depart from normal, the probability p of such departure on a Chi-square test being 0.097. (On the next group that I tried the p turned out to be 0.250.)

The mean of this thousand totals is 32,164,000 (estimated with standard deviation of pure sampling variation of 57,349) and their standard deviation of forecast is 1,814,000 estimated with sampling S.D. of 40,456. The first and 99th percentiles of the distribution that came out of the particular simulation are 27,945,000 and 36,383,000 respectively, and the 5th and 95th are 29,181,000 and 35,147,000.

HOW FAR IN THE FUTURE IS THE RESULT USABLE?

The conclusion to be drawn from Table 2 and the numbers given earlier is that the estimate for 2020 could well be too uncertain to be useful. The utility of projections to 40 years from the jumping off date is at least put in question. Hence we are interested in similar results for nearer horizons. Table 3 shows the distributions for 1985 to 2020, all projected from 1980. We could have been 99 percent sure that the 1985 population will fall between 25 and 26 million, and 85 percent sure that the 2000 population would fall between 28 and 30 million. That for 2000 about 85 per cent of the probability is concentrated between 28 and 30 millions contrasts with the distribution for 2020, where it takes a range of 30 to 35 million to embrace 85 percent of the distribution.

Table 3. Distribution of totals for 1985 to 2020, all projected from 1980.

Interval	1985	1990	1995	2000	2005	2010	2015	2020
23000	0	0	0	0	0	0	0	0
24000	2	0	0	0	0	0	0	0
25000	991	6	0	0	0	0	0	0
26000	7	630	25	1	0	1	1	2
27000	0	363	406	62	23	9	10	9
28000	0	1	528	373	134	57	34	28
29000	0	0	41	440	354	167	110	79
30000	0	0	0	90	353	334	171	113
31000	0	0	0	3	122	270	281	197
32000	0	0	0	0	14	128	216	218
33000	0	0	0	0	0	30	117	176
34000	0	0	0	0	0	4	47	102
35000	0	0	0	0	0	0	11	53
36000	0	0	0	0	0	0	2	17
37000	0	0	0	0	0	0	0	5
38000	0	0	0	0	0	0	0	1
39000	0	0	0	0	0	0	0	0

While there cannot be any definite point of time beyond which the projection becomes unusable, yet it may be desirable to set up a convention on the matter. Statisticians long ago established a convention that an experimental result is significant when it has a 0.05 chance or less of emerging when the null hypothesis is true. That could equally well have been 0.15 or 0.025, yet it has been convenient to apply a standard uniformly, however arbitrary. R.A. Fisher, to whom the 0.05 seems to have been due, said that if of 20 scientific papers only one incorrectly announced a positive result when the null hypothesis really held, science would be doing remarkably well.

For projections it may similarly be convenient to set a standard, for example a maximum acceptable standard error of 6 per cent of the forecast. If the chance of the outcome being within 6 per cent of the forecast is 2/3, say, then the result is useful; with larger error than that it ought not to be published. I doubt if such a convention will come into force in the way that the famous 0.05 has come to be standard for tests of significance, but it is mentioned as a possibility.

Judging from the distributions of Table 3 such a convention would mean that for total projections to 2005 or 2010 (say 25 to 30 years ahead) are worthwhile, and those beyond are not worth publishing. This also was the conclusion of Cohen (1986, p.122). Some impressions on such matters is to be had from the two figures, which represent alternative ways of using the past data, and show approximately the same. *ex ante* error. Figure 2 can be regarded as a smoothed version of Fig. 1.

DISTRIBUTION OF THE 2020 RATIO OF 65+ TO 20-64

The series of random projections tells a different story for ratios among age groups, taken for our example as that of the persons 65 years and over to those 20 through 64. From some points of view this is more important than the total population, since it raises social security issues that are more sensitive than the question how many people there will be than the mere total. Also, whether we like it or not, the projection at least to 2020 is unavoidable, since it is only shortly before then—when the baby boom of the 1950s is at retirement age—that the difficulties of social security will set in and on which public attention is now concentrated.

Table 4 shows the observed to have a bimodal distribution, with the first 100 values of the thousand as arranged in order being a separate more or less normal distribution, distinct from the approximately normal distribution of the last 900. For these 900 the Chi-square test of goodness of fit came out to 15.9 with 8 degrees of freedom, or a probability p of 0.045, just on the edge of a significant departure from normality, not to be taken seriously. There is no question of the departure of the whole from normality, so the fitting in the second column cannot be regarded as appropriate or useful in any way. Just to make sure that the bimodality was real I tried a second thousand, as shown, and to the last 900 items fitted a normal, with essentially the same result.

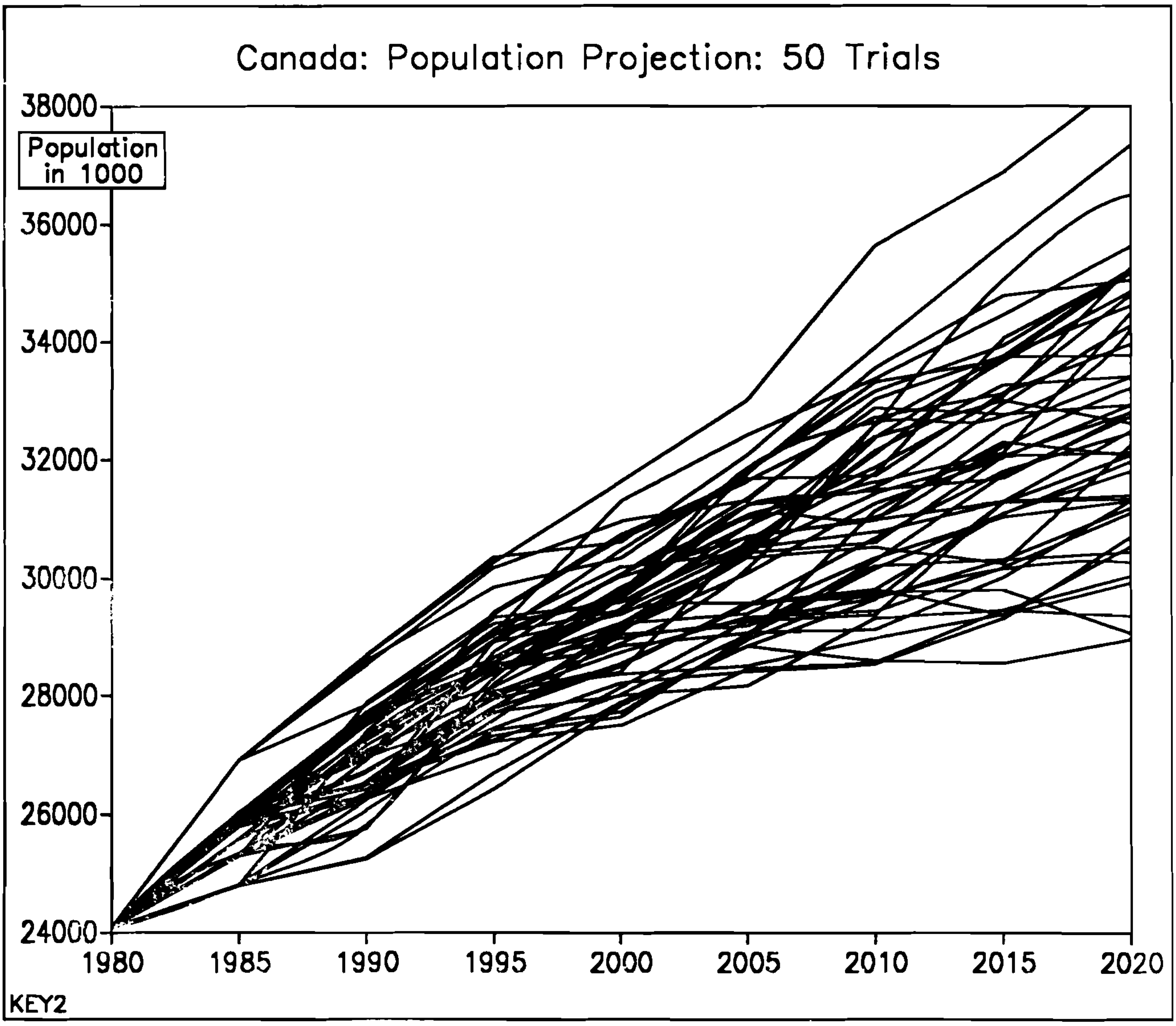

Figure 1. Projection made by selecting a fresh random past year for inputs in each five-year cycle.

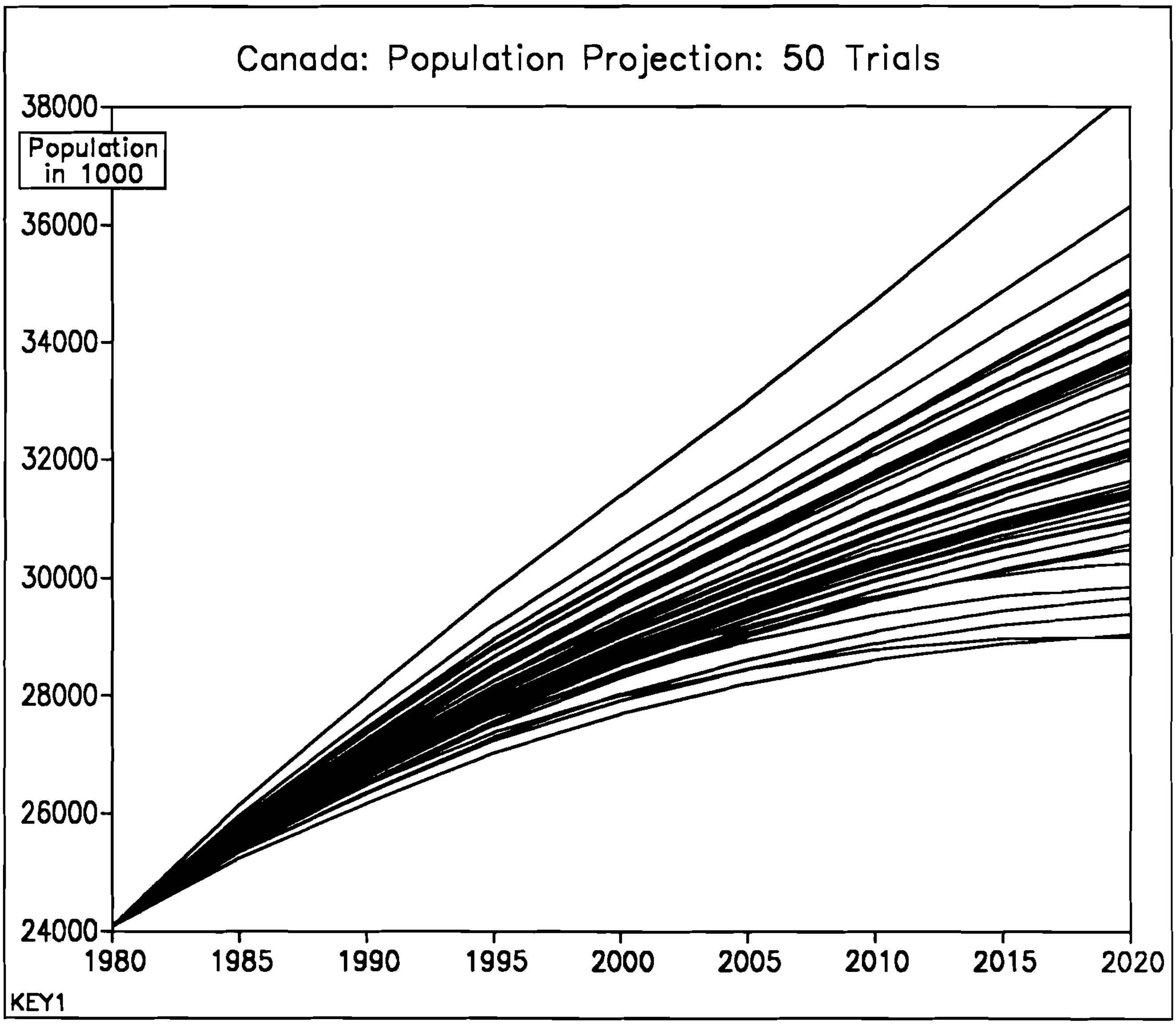

Figure 2. Projections made by selecting nine earlier years at random without replacement, averaging them, and using the average input for all future years.

Table 4. Frequencies of ratios of persons 65 and over to those 20-64 for the 2020 population total when birth, mortality improvement, and migration are selected at random from past experience.

Ratio	First thousand Obs.	First thousand Fitted Normal	Second thousand Observed	Second thousand Normal Fitted to 0.25 and above
0.21-	14	3	13	
0.22-	42	8	47	
0.23-	37	18	26	
0.24-	6	36	12	
0.25-	21	62	16	15
0.26-	56	94	54	43
0.27-	86	123	77	93
0.28-	141	142	112	152
0.29-	189	144	205	188
0.30-	193	127	192	176
0.31-	118	98	134	124
0.32-	57	67	75	66
0.33-	29	40	25	27
0.34-	9	21	10	8
0.35-	1	9	2	2
0.36-	1	6	0	0

DECOMPOSITION OF THE UNCERTAINTY

How much of the variation is due to uncertainty on births, how much to deaths, how much to migration? Which of the three components, birth, death, or migration, cause the unexpected bimodality in the ratio of the 65+ to the 20–64? It is easy to decompose the overall variation into the parts due to the three input components. The method is merely to do three sets of trials essentially the same as the one reported above, but for one of which births only are selected by the random process described, for one deaths, for one migration.

EFFECT OF VARIATION OF THE FERTILITY COMPONENT ALONE

We start with fertility alone varying, while mortality improvement and migration are held at their average values of the preceding 60 years (Table 5).

Table 5. Mean and standard deviation for total population and for ratio of 65+ to 20–64 when fertility alone varies.

Mean and standard deviation (x 1000) for total population

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean	24089	25608	26902	28131	29165	30088	30983	31829	32591
SD	0	153	308	456	602	770	987	1247	1523

Mean and standard deviation (x 1000) for ratio of 65+ to 20-64

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean	166	177	186	198	205	210	227	265	309
SD	0	0	0	0	0	2	4	6	10

The standard deviation measuring uncertainty in the output total is very close to what we found with all three components varying. The standard deviation for the total in 2020 is 1,523,000 with fertility alone varying, against 1,849,000 when all three components vary, i.e. about 5/6 of the overall uncertainty of the total population is due to ignorance of what past level the births will follow. But for the ratio 65+/20-64 by 2020 births account for only 0.010 out of a total 0.027.

MORTALITY

We go on to suppose that only mortality improvement is subject to variation, which is to say that we know exactly fertility and migration, and their amounts are the average of the preceding 60 years. Then we obtain the numbers shown in Table 6.

Table 6. Mean and standard deviation for total population and for ratio of 65+ to 20–64 when mortality alone varies.

Mean and standard deviation (x 1000) for total

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean	24089	25600	26876	28078	29077	29955	30793	31563	32229
SD	0	25	76	149	243	361	500	664	856

Mean and standard deviation (x 1000) for ratio of 65+ to 20-64

Mean	166	177	184	195	199	202	217	251	291
SD	0	1	2	5	8	11	14	19	25

This gives a standard deviation of uncertainty by 2020 of 856,000 in the total, but fully 0.025 (out of 0.027) in the ratio—i.e. by far the largest part of the uncertainty in the 2020 ratio is due to uncertainty on mortality improvement.

MIGRATION

With migration only uncertain we obtain Table 7.

Table 7. Mean and standard deviation for total population and for ratio of 65+ to 20–64 when migration alone varies.

Mean and standard deviation (x 1000) for total

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean	24089	25604	26894	28118	29146	30062	30946	31778	32525
SD	0	66	138	215	296	382	472	565	662

Mean and standard deviation (x 1000) for ratio of 65+ to 20-64

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Mean	166	177	186	199	205	210	228	265	309
SD	0	0	1	1	1	2	2	3	4

Evidently of the three inputs migration is the least responsible for future uncertainty. Presumably this is because the age distribution of migrants has tended to approach that of the population as a whole.

INTERACTIONS

Table 8 shows a calculation for a fresh 1000, and follows this with a summary in matrix form for 2020.

Table 8. Standard deviations for variation due to separate components.

	1980	1985	1990	1995	2000	2005	2010	2015	2020
All three components selected at random									
S.D. for all	0	170	349	532	723	946	1221	1543	1894
Only births selected at random									
S.D. for births	0	153	308	456	601	769	985	1243	1516
Only mortality selected at random									
S.D. for mort	0	24	73	144	235	348	483	640	823
Only migration selected at random									
S.D. for mig	0	62	130	203	280	361	446	534	625

or rearranged for 2020, and including variation of two inputs at a time:

	Fertility	Mortality	Migration	
Fertility	1516	1658	1594	
Mortality		823	1101	
Migration			625	
All together				1894

Among the diagonals, in the bottom section of the table, that show the pure effects of each of the inputs, the results for this 1000 are essentially what we saw earlier. The births account for the largest part of the standard deviation in the estimate of the total. In terms of variances, the square root of the sum of squares of the effects of the three components singly for 2020 is 1835, somewhat less than the effect of the three together at 1894, so there is a trifling positive interaction.

In order to study the interactions of the three elements we need to allow two to vary at a time. That is done in the off-diagonal elements of Table 8, and in Table 9, but showing only the standard deviations. Of course it would have been possible to make a choice at random for a past year, or five-year period, and for that year take all three components, and in that case a correlation over time between births and deaths, say, would show up as interaction. With only 13 points of time at 5-year intervals it would not be expected to give the relatively smooth distributions of Tables 2 to 4. Since correlations among the three inputs are in fact low the overall result would have been much the same as we have found.

Table 9. Standard deviation for total and for ratio of 65+ to 20–64 when two components vary at a time.

FERTILITY AND MORTALITY VARYING

	1980	1985	1990	1995	2000	2005	2010	2015	2020
standard deviation (x 1000) for total	0	152	308	463	623	813	1055	1344	1658
standard deviation (x 1000) for ratio of 65+ to 20-64	0	1	2	5	8	11	15	20	27

MORTALITY AND MIGRATION VARYING

	1980	1985	1990	1995	2000	2005	2010	2015	2020
standard deviation (x 1000) for total	0	71	158	264	388	533	699	887	1101
standard deviation (x 1000) for ratio of 65+ to 20-64	0	1	3	5	8	11	14	19	26

MIGRATION AND FERTILITY VARYING

	1980	1985	1990	1995	2000	2005	2010	2015	2020
standard deviation (x 1000) for total	0	160	324	484	643	824	1049	1314	1594
standard deviation (x 1000) for ratio of 65+ to 20-64	0	0	1	1	1	2	4	7	10

Apparently there is some positive interaction, but it is small. That is confirmed by a further set of trials, where the square root of the diagonal items corresponding to the matrix in Table 9 was 1849, while the SD when all three act together was 1885.

Table 10. Distribution of uncertainty for 2020 in forecast total population when all components vary, and part of the uncertainty when one component only varies.

	all	births	deaths	migration
27,000,000-	0	0	0	0
28,000,000-	7	0	0	0
29,000,000-	31	1	0	0
30,000,000-	68	32	89	0
31,000,000-	117	114	0	1
32,000,000-	208	196	159	182
33,000,000-	217	260	665	550
34,000,000-	174	211	87	256
35,000,000-	108	115	0	11
36,000,000-	48	56	0	0
37,000,000-	19	14	0	0
38,000,000-	3	1	0	0
39,000,000-		0	0	0
	1000	1000	1000	1000

Table 10 shows for total population in 2020 the actual distributions when one component varies at a time, and Table 11 shows the same for the ratio of the 65+ to the 20–64. The irregularity of the groups in Table 10, and even more in Table 11, arises because we are using only 13 past points of time as data, at five-year intervals from 1921 to 1981. One could make the results smoother by recognizing the 61 single years, and choosing at random among them. Evidently it is the bimodality of deaths, rather than of the other two components, that underlay the bimodality in Table 4.

Table 11 shows how mortality has the largest effect on the uncertainty of the ratio, followed by fertility, followed by migration. Migration does vary more from year to year than the other two, but on the other hand it is a less important element in population growth, and in the ratio of age groups.

Table 11. Distribution of uncertainty for 2020 in ratio of 65+ to 20-64 when all components vary, and the part of the uncertainty when one component only varies.

	all	births	deaths	migration
0.210-	0	0	0	0
0.220-	13	0	0	0
0.230-	47	0	76	0
0.240-	27	0	0	0
0.250-	11	0	0	0
0.260-	16	0	0	0
0.270-	56	0	89	0
0.280-	77	0	79	0
0.290-	111	21	0	0
0.300-	206	152	291	12
0.310-	192	361	368	617
0.320-	136	334	0	371
0.330-	72	120	97	0
0.340-	24	12	0	0
0.350-	10	0	0	0
0.360-	2	0	0	0
0.370-	0	0	0	0
Total	1000	1000	1000	1000

ERROR OF ENUMERATION AND FORECASTING ERROR

Since very different considerations apply to the error of the census and other data for the jumping off point, on the one hand, and the error of forecasting on the other, it is well to separate these sources in the analysis. In the present context that means we disregard all errors in data referring to a past period. We calculate as though censuses and vital statistics are exact; without asserting that they are exact, we consider it helpful to separate our enemies, and combat enumeration errors and projection errors with different weapons.

If there is no error at the jumping off point then we may apply a principle of continuity and take it that there is little error in the year following, and only somewhat more error in the year after that. How fast the error increases into the future is discussable in terms of the form of the lines bounding the projection. If the error is just proportional to the elapsed time from the last data, then we would have straight lines bounding the possibilities. If the error increases with time faster than proportionally we will have a horn shape; if less than proportional a lily shape. It is not easy to establish this shape empirically; part of what emerge in any such work as the present is a function of the hypotheses underlying the calculation.

COMPARISON WITH ACCURACY OBSERVED TO HAVE BEEN ATTAINED IN PREVIOUS FORECASTS

In previous work upwards of 1000 forecasts (Keyfitz, 1981) for some 60 countries, made between about 1955 and 1975, were compared with the population that subsequently materialized. One way of describing the result is in terms of the implied rate of increase from the jumping-off time to the point being forecast; it was found that for developed countries the standard error of forecasts of the implied rate of increase was about 0.2 percentage points. There was some tendency to improvement in the forecasts as time went on, so let us take it that the *ex post* standard error of estimate of the rate of increase in percentage is 0.14 percentage points.

Let us apply this way of considering projections in terms of the angle of climb foreseen for the population. If the 2020 total is to be 32,211,000 as in the average for the second sample above, that implies a certain average rate of increase over the forty years 1980–2020, i.e. a mean annual increase of 0.73 per cent and a standard deviation of 0.14 percentage points. This is to say that rate at which the population is expected to grow, of 0.73 per cent per year, is subject to a standard deviation of 0.14, i.e. we should be able to bet 2 to 1 odds that the average rate will fall between 0.59 and 0.87 per cent per year, so that the 2020 figure will fall between 24089exp(40*0.0059) and 24089exp(40*.0087). This is with 2 to 1 odds; for 19 to 1 odds we would double the range. Such statements depend on the errors being normally distributed, a condition shown to hold fairly well.

The comparison with the *ex ante* of this article is shown in Table 12.

Table 12. Comparison of *ex ante* and *ex post* calculation of standard deviation of uncertainty.

	1980	1985	1990	1995	2000	2005	2010	2015	2020
Ex ante	0	168	344	523	710	929	1200	1518	1863
Ex post	0	187	389	606	838	1087	1354	1640	1945

CONCLUSION

Thus the *ex ante*, as calculated by the crude methods of this article, shows slightly less error than the *ex post*, i.e. than the errors to which past forecasts have been subject. This result presents a puzzle: How could a method that selects each future year's fertility at random from the past fail to give a much-too-high variance? The answer can only be that we know there will be change, but not knowing in what direction we either preserve the present into the future unaltered, or else they forecast up and down, or down and up, but essentially revolving around the past values..

Inspection of individual past forecasts shows a sensitivity on the part of forecasters to what is going on around them at the moment of making the calculation. When they see a baby boom they take that as the future level. There is just not enough regularity in births, mortality changes, and migration to enable the forecaster to see the moment as the top of a cycle, or bottom. Nor would extrapolation of past trends be any improvement. Ronald Lee (1974) has shown that gains are possible by taking account of serial correlation, in particular for births, but that has not been included in the present series of tests.

The present calculation, initially undertaken merely to provide an *ex ante* estimate of the error of a forecast, turns out to do somewhat more than that, to constitute an interpretation of forecasts as they have been made over the past 40 years.

APPENDIX: RESULTS OF A LARGE-SCALE SIMULATION

Assuming as before 1) that the uncertainty of population forecasts is determined by the uncertainty of the three inputs; 2) that the uncertainty of a variable depends on its variability, and 3) that future variability in the inputs will be about the same as past variability. Our inputs represent the historical series for the preceding 60 years (at 5 year intervals), and we can both find the overall uncertainty and also decompose it, i.e., determine the parts arising out of each of the three inputs.

In all, 32,000 trials are reported here. In one half of these a single year was selected from the historical record and for that year the values were chosen for all three inputs, and used for all 8 five-year cycles of projection from 1980 to 2020: in one half a fresh random choice was made for each five-year cycle. Similarly for each of two other contrasts in a factorial design. The experimenting covered all combinations of three two-valued variables:

- whether the same year was used for all cycles of projection or an independent selection made for each cycle;
- whether the same year was used for the three inputs or the year for each input was selected independently;
- number of years selected from the past and averaged, whether 1 or 9;

The arrangement being factorial, for any contrast of these two-valued variables we can compare 16,000 trials on the one side with 16,000 on the other. That will be done later in Tables A12 to A14. First we show results for all trials together with no contrasts, breaking down the results according to which of the inputs was allowed to vary. In short we assumed ignorance of the future of one of the inputs only (Table A1).

Table 1A. Average of all 32,000 trials: standard deviation of the forecast total population in thousands of persons

		With ignorance of		
Year	all 3	birth only	death only	mig. only
	(1)	(2)	(3)	(4)
1990	647	557	39	273
2000	1148	956	116	516
2010	1787	1483	231	770
2020	2643	2225	392	1035

In round numbers, the first column of Table A1 tells us that for the year 2000 we can know the population to within 1.1 million persons, and by 2020 within 2.6 million, both with a certainty of 2/3. Multiplying these by two gives the uncertainty with the higher standard of probability 95%.

Table A1 shows that on our procedure the increase in uncertainty with time is more than exponential for the longer projections spans. Thus the value for 2000 (projected from 1980 this is a 20-year span) is slightly less than twice that for 1990 (a 10-year span), but that for 2020 is considerably more than double 2000.

If we could foresee exactly the future mortality and migration but were ignorant of future fertility, we would by 2020 still be subject to a standard deviation of 2.2 million or 84 per cent of the error with all three inputs unknown. (Column 2 of Table A1.) Mortality alone would give a standard error of 0.4 million or 15 per cent, and migration alone of 1.0 million or 39 per cent. The square root of the sum of the squares of these percentages is 94.0 (Table A2). Apparently the three sources or error are not independent, but are positively related. It is the absolute effect on the projection that are measured, and when the births are high there will be more people, so with given death rates there will then be more deaths.

Table A2. Percent of effect due to the several inputs on average of all 32,000 trials

	Ignorance of				
Year	all	birth	death	mig.	RSS of three inputs
1990	100.0	86.0	6.0	42.2	96.0
2000	100.0	83.3	10.1	44.9	95.2
2010	100.0	83.0	12.9	43.1	94.4
2020	100.0	84.2	14.8	39.2	94.0

Though for short term projections mortality has little effect it contributes considerable error over a 40 year projection. On the other hand, migration tends to fall off relatively for the long term forecasts.

We can follow up this matter further by considering the inputs two at a time, and see in what degree the errors of one increase the errors of the other.

Table A3. Ignorance of inputs two at a time--all trials

	ignorance of		
Year	death+mig.	birth+death	birth+mig.
1990	280	559	637
2000	542	968	1119
2010	830	1519	1725
2020	1154	2296	2526

Percentages of standard deviation for all three inputs as shown in Table A1.

	ignorance of		
Year	death+mig.	birth+death	birth+mig.
1990	43.3	86.4	98.5
2000	47.2	84.3	97.5
2010	46.5	85.0	96.5
2020	43.7	86.8	95.6

Thus if we knew birth and migration we would eliminate 95.6 per cent of the uncertainty that exists by 2020. How is this to be reconciled with Table A2, in which it appears that if we are ignorant of mortality only we would be subject to 14.8 per cent of the standard deviation of the total error? Of course it is not proper to add 94.5 and 14.8; we must take the square root of the sum of their squares, which is 95.7.

We would not expect this to come out exactly to 100.00 per cent because the several columns of the table are based on separate simulations, so there is random error in the comparisons. But given that each of the columns is based on many thousands of repetitions of the projection, this random element should not be large. What is more important is that when birth and migration work together we do indeed include the positive interaction between them, but we do not include the interaction of both of them with births. Presumably that accounts for most of the missing 3.3 per cent.

Table A4 has subtracted the appropriate sum of squares from the items of Table A2 and A3, and so gives the interaction, insofar as it appears above the noise constituted by sampling error. That interaction is in all cases small and positive. Thus by 2020 the error due to birth and migration together is 2.8 per cent of the whole error more than is to be expected from the sum of the errors due to births and migration separately. The sum of this plus the similar errors in the other two pairs is about equal to the interaction of all three of the inputs.

Table A4. Net amount of interaction in forecasting totals for the average of all trials. Interactions as per cent of the standard deviation when all inputs are uncertain, and when two of the three inputs are uncertain.

		ignorance of		
Year	all	death+mig.	birth+death	birth+mig.
1990	4.0	0.7	0.2	2.7
2000	4.8	1.2	0.4	2.9
2010	5.6	1.5	1.0	3.0
2020	6.0	1.8	1.4	2.8

But we are also interested in the relation of these standard errors to the projected population. Dividing the numbers in Table A1 above by mean projected population (about 33 million by 2020) gives the overall coefficient of variation, in percentages (Table A5).

Table A5. Percent coefficient of variation of overall error and of the error due to ignorance of birth, death, and migration respectively: total population.

		ignorance of		
Year	all	birth	death	mig.
1990	2.39	2.06	0.14	1.01
2000	3.91	3.26	0.39	1.76
2010	5.71	4.74	0.74	2.46
2020	8.01	6.74	1.19	3.14

The conclusion (first column) is that by the year 2000 we will know the total population to within about 4 per cent, and by the year 2020 to within about 8 per cent, both with probability 2/3.

Note that we cannot say whether this is good or bad, but only that if uncertainty on the inputs (birth, death migration) is equal to the amount of variation that these will show, and if the future variation is to be the same as the past, then this is what we will get on the average of a number of ways of doing the projection.

As late as the 1950s demographers forecast a population of 3 billion for the world by the end of the century, and now we know as virtually certain that it will be more than double that--a 100 per cent error. On the other hand in the early 1970s we read demographers forecasting 6.4 billion for 2000, and now 6.2 billion is thought likely, an error of only 3 per cent. By looking at the one kind of example we could say "Demographers simply cannot forecast the future population with useful accuracy." or "Demographic forecasts are remarkable accurate, especially if compared with forecasts of income, technology, resources, or other variables.

To this writer neither of these two statements is of any value. We do not know whether we are the lucky forecasters or the unlucky ones, and should rather think in terms of a quantitatively measured expected degree of ignorance. This *ex ante* estimate of error, made with the same data that produces the forecast itself, can be checked against the *ex post* errors, that a collection of forecasts have actually made in the past, and when this is done it turns out that the two are in agreement.

ERROR OF RATIOS

All of the above concerns the future total population. We go on to discuss errors of ratios, and use as an example the error of the population 65 years of age and over divided by the population 20 to 64 years of age, i.e. roughly speaking the retired ages over the working ages. The ratio was calculated for each of the 32,000 projections, and the standard error of the distributions measured, again with ignorance of all three inputs, and with ignorance of one at a time. (Table A6)

Table A6. Standard deviation of the forecast ratio of 65+ divided by the 20-64, from 32,000 projections, with ignorance of the three inputs respectively

	Ignorance of future			
Year	all	birth	death	mig.
1990	0.0018	0.0000	0.0013	0.0013
2000	0.0042	0.0000	0.0036	0.0025
2010	0.0098	0.0065	0.0064	0.0034
2020	0.0199	0.0150	0.0114	0.0062

It is apparent that birth is the chief source of error for the long term forecasts, and (as one could see without doing the simulation) not the source of any error at all for the first 20 years. Death is about four times as important here as it is in the total population, while migration is only slightly more important. In percentage terms we have Table A7 corresponding to Table A2.

Table A7. The percentage impact of ignorance of the three inputs severally on the standard deviation of projection.

	Ignorance of future			
Year	all	birth	death	mig.
1990	100.0	0.0	72.8	73.9
2000	100.0	0.0	85.2	58.4
2010	100.0	66.3	65.0	35.1
2020	100.0	75.6	57.4	31.1

Corresponding to Table A4 we have the coefficients of variation of Table A8, again the numbers in Table A6 divided by the mean values of the projection to future times. Apparently the ratios have a smaller coefficient of variation than do the totals as shown in Table A5, and this applies separately to the ignorance of the future caused by birth and migration, though not for death.

Table A8. Percent coefficient of variation of overall error and of the error due to ignorance of birth, death, and migration respectively: ratio of 65+ to 20-64, for all trials.

	ignorance of			
Year	all	birth	death	mig.
1990	0.95	0.00	0.69	0.70
2000	2.11	0.00	1.80	1.23
2010	4.55	3.02	2.96	1.60
2020	6.94	5.25	3.99	2.16

Following (Table A9) are the interactions of the ratios, still taken from the 32,000 trials. That for death plus migration for example is calculated by subtracting the variance due to death alone plus that due to migration from that due to both simultaneously, then taking the square root and multiplying by 100.

Table A9. Net amount of interaction in forecasting totals for the average of all trials. Interactions as per cent of the standard deviation when all inputs are uncertain, and when two of the three inputs are uncertain.

		ignorance of		
Year	all	death+mig.	birth+death	birth+mig.
1990.0	-3.7	-3.2	0.4	0.7
2000.0	-3.3	-3.4	0.3	0.2
2010.0	0.7	-2.4	1.4	2.0
2020.0	0.1	-2.1	-0.5	1.8

MOST LIKELY CASE

The above results derived from the entire set of 32,000 simulations probably has too wide a scope. For example one of the sets included a random amount of birth, improvement in mortality, and migration, chosen once and for all from some past year, and then used for the entire projection from 1980 to 2020. That would seem to give too large a variation.

In what one may think of as the most likely case a separate random past year was chosen and its birth rates, mortality improvement, and migration, applied to the first cycle of projection--1980 to 1985--then a fresh random year selected for the next cycle, and so on. In any one cycle all inputs came from the same past year. There was no averaging of past values in making each individual projection. Of the 8 possibilities in the factorial design this was one, and it had 4,000 trials.

Table A10. Most likely case: One past random year chosen for each cycle of projection, providing all three inputs. Standard deviation of total population and ratio 65+/20-64.

	Uncertainty on			
Year	all	birth	death	mig.
	Total population			
1990	823	694	51	340
2000	1200	950	135	517
2010	1642	1293	256	673
2020	2207	1778	415	819
	Ratio 65+/20-64			
1990	0.0022	0.0000	0.0017	0.0016
2000	0.0046	0.0000	0.0041	0.0025
2010	0.0106	0.0081	0.0070	0.0031
2020	0.0190	0.0149	0.0122	0.0050

Dividing the numbers in Table A10 by means gives the coefficient of variation for the most likely case in which one past year is selected for each cycle, all cycles are separately drawn and three inputs come from the same past year. (Table A11)

Table A11. Most likely case: One past random year chosen for each cycle of projection, providing all three inputs. Per cent coefficient of variation of forecast total population and ratio 65+/20-64.

		Ignorance of		
	all	birth	death	mig.
		Total population		
1990	3.05	2.57	0.19	1.26
2000	4.09	3.23	0.46	1.76
2010	5.25	4.14	0.82	2.15
2020	6.69	5.39	1.26	2.48
	Age 65+/20-64			
1990	1.17	0.00	0.92	0.87
2000	2.31	0.00	2.07	1.24
2010	4.92	3.76	3.27	1.42
2020	6.64	5.20	4.26	1.75

The coefficients of variation are somewhat smaller than when all the data are used, as in Tables A5 and A8, but not as much smaller as this writer expected.

CONTRASTS OF METHODS.

A part of the purpose of drawing the 32,000 projections was to see what differences in the variation would be associated with different ways of projection, or more strictly with different ways of drawing the inputs.

Table A12. Inputs fixed through cycles, vs. changed with cycle

		ignorance of		
Year	all	birth	death	mig.
		Total population		
1990	1.41	1.42	1.30	1.42
2000	2.00	2.00	1.61	2.00
2010	2.42	2.42	1.78	2.44
2020	2.74	2.73	1.87	2.81
		Ratio 65+/20-64		
1990	1.36	-	1.31	1.44
2000	1.71	-	1.65	2.01
2010	1.77	1.42	1.81	2.40
2020	2.16	2.01	1.90	2.72

Table A12 gives the ratio of standard error when we compare the projection choosing a past year, and using its fertility, mortality improvement, and migration for all future years, on the one side, with choosing afresh for each five-year cycle of projection on the other. The former of course gives higher variation, and Table A12 shows in what ratio. Since the former method involves the choice of 9 random numbers, and the latter only one, we might have expected that the standard deviations would be in the ratio of 3 to 1, but in fact they are considerably less.

Table A13. Individual past years versus average of 9 past years:

		ignorance of		
Year	all	birth	death	mig.
		Total population		
1990	3.00	2.99	3.08	3.01
2000	3.00	2.98	3.12	3.00
2010	3.02	2.98	3.17	3.00
2020	3.03	3.00	3.20	3.00
		Ratio 65+/20-64		
1990	3.06	-	3.18	3.02
2000	3.11	-	3.20	2.98
2010	3.10	3.00	3.27	2.95
2020	3.16	3.00	3.34	2.95

However Table A13 does show ratios of very nearly 3 to 1. In it we are comparing the projections with the choice of a single past year for the input to each projection, versus the choice of 9 past years and averaging.

Table A14. Fixed for all 3 inputs, versus independent for inputs:

Year	all	Ignorance of birth	death	mig.
		Total population		
1990	1.09	1.00	0.99	1.00
2000	1.11	1.00	0.99	1.00
2010	1.12	0.99	0.99	1.00
2020	1.12	0.99	0.98	1.00
		Ratio 65+/20-64		
1990	0.94	-	0.99	0.99
2000	0.95	-	0.99	1.00
2010	1.08	1.00	0.99	1.00
2020	1.09	0.99	0.98	1.00

Table 14 compares the choice of a single past year for all three inputs with a fresh choice of past year for each of fertility level, mortality improvement, and migration. The former is slightly higher (at least for the totals) as one would expect if years that are high in fertility are also high in mortality improvement and in migration.

References

Cohen, Joel E. (1986) Population forecasts and confidence intervals for Sweden: A comparison of model-based and empirical approaches. *Demography* 23:105–126.

Henry, L. and H. Gutierrez (1977) Qualité des prévisions démographiques à court terme. Étude de l'extrapolation de la population totale des départements et villes de France, 1821–1975. *Population* (Paris) 32:625–647.

Keyfitz, Nathan (1981) The limits of population forecasting. *Population and Development Review* 7(4):579–593.

Land, K.C. (1985) *Methods for National Population Forecasts: A Critical Review.* Population Research Center Paper 7.001, University of Texas at Austin.

Lee, R.D. (1974) Forecasting births in post-transition populations: stochastic renewal with serially correlated fertility. *Journal of the American Statistical Association* 69:607–617.

Spencer, Bruce D. and Juha M. Alho (1985) Uncertain population forecasting. *Journal of the American Statistical Association* 80(390):306–314.

Stoto, M.A. (1983) The accuracy of population projections. *Journal of the American Statistical Association* 78:13–20.

Tuljapurkar, Shripad (1989) An uncertain life: Demography in random environments. *Theoretical Population Biology* 35:227–294.

Gaussprogramme that generated the numbers summarized in the Appendix tables.

```
@LOOP FOR SELECTING RANDOM MORTALITY@
                        @apply random selection from the 12 sets of values of Lx@
load shorlife;lll=shorlife;       @shorlife is the record of life table Lx@
k2=int(rndu(f,1)*11+1); @for selecting new random past year for mortality@
if jjjjj==1;k2=k1;endif;                @for same random year for all inputs@
if jjj==4;let k2=10;endif;
if jjj==5;let k2=10;endif;
if jjj==7;let k2=10;endif;
if jjj==8;let k2=10;endif;
                                        @ This removes the variation in deaths@
rr=reshape(shorlife,13,19*9);llx=reshape(meanc(rr[k2,.]),19,9);
ppx=trim(llx,1,0)./trim(llx,0,1);       @ppx is the ratio of successive Lx@

@LOOP FOR SELECTING RANDOM NET MIGRATION@
k3=int(rndu(f,1)*11+1);                 @for selecting f new random past years@
if jjjjj==1;k3=k1;endif;                     @same random years for all inputs@
let mmm=72 64 125 95 73 194 164 168 154 71 74 100;
    if jjj==3;mmm=ones(12,1)*meanc(mmm);endif;
    if jjj==5;mmm=ones(12,1)*meanc(mmm);endif;
    if jjj==6;mmm=ones(12,1)*meanc(mmm);endif;
    if jjj==8;mmm=ones(12,1)*meanc(mmm);endif;
      @mmm=ones(12,1)*meanc(mmm); This removes the variation in migration@
mmm=(mmm-40)*5;              @future migration less than historic by 40,000@
imx=ddd[26:43,.];immmx=(imx/sumc(imx))*meanc(mmm[k3,.]);

@CARRY OUT THE PROJECTION@
  @ j=0;do while  j<e-1;j=j+1;           AAAAAA--j is the projection cycle@
                 @comment out if fresh choice is to be made in each cycles@
     px=ppx[.,j];px=trim(px,0,1);
     trun1=px.*trim(pp[.,j],0,1);trun2=(trun1+trim(pp[.,j],1,0))/2;
     fertpop=trun2[3:9,.];births=bbx'fertpop*llx[1,j]/2;
     pp[.,j+1]=(births|trun1)+immmx;
     endo;           @pp contains the projected population for jjth trial@

@VARIOUS SUMMARIES FOR jjTH TRIAL AND SAVING OF RESULTS FOR ALL TRIALS TOGETHER@
y=sumc(pp)';            @y is the horizontal vector for the year by year totals@
if int(jj/600)*600==jj;y;endif;               @shows every 100th trial on screen@
outt[jj,.]=y;                @saves up year by year totals for successive trials@
r=rows(pp);ratio=sumc(pp[14:r,.])./sumc(pp[5:13,.]);
outr[jj,.]=ratio';                         @saves up year by year ratios of 65+/20-64@

endo;                                @ends the jj loop for the successive trials@
mt=meanc(outt)';st=stdc(outt)';
mr=meanc(outr)';sr=stdc(outr)';

if jjj==1;rt1[.,1]=st[.,3];rr1[.,1]=sr[.,3];endif;
if jjj==2;rt1[.,2]=st[.,3];rr1[.,2]=sr[.,3];endif;
if jjj==3;rt1[.,3]=st[.,3];rr1[.,3]=sr[.,3];endif;
if jjj==4;rt1[.,4]=st[.,3];rr1[.,4]=sr[.,3];endif;
if jjj==5;rt1[.,5]=st[.,3];rr1[.,5]=sr[.,3];endif;
if jjj==6;rt1[.,6]=st[.,3];rr1[.,6]=sr[.,3];endif;
if jjj==7;rt1[.,7]=st[.,3];rr1[.,7]=sr[.,3];endif;
```

```
if jjj==1;rt2[.,1]=st[.,5];rr2[.,1]=sr[.,5];endif;
if jjj==2;rt2[.,2]=st[.,5];rr2[.,2]=sr[.,5];endif;
if jjj==3;rt2[.,3]=st[.,5];rr2[.,3]=sr[.,5];endif;
if jjj==4;rt2[.,4]=st[.,5];rr2[.,4]=sr[.,5];endif;
if jjj==5;rt2[.,5]=st[.,5];rr2[.,5]=sr[.,5];endif;
if jjj==6;rt2[.,6]=st[.,5];rr2[.,6]=sr[.,5];endif;
if jjj==7;rt2[.,7]=st[.,5];rr2[.,7]=sr[.,5];endif;

if jjj==1;rt3[.,1]=st[.,7];rr3[.,1]=sr[.,7];endif;
if jjj==2;rt3[.,2]=st[.,7];rr3[.,2]=sr[.,7];endif;
if jjj==3;rt3[.,3]=st[.,7];rr3[.,3]=sr[.,7];endif;
if jjj==4;rt3[.,4]=st[.,7];rr3[.,4]=sr[.,7];endif;
if jjj==5;rt3[.,5]=st[.,7];rr3[.,5]=sr[.,7];endif;
if jjj==6;rt3[.,6]=st[.,7];rr3[.,6]=sr[.,7];endif;
if jjj==7;rt3[.,7]=st[.,7];rr3[.,7]=sr[.,7];endif;

if jjj==1;rt4[.,1]=st[.,9];rr4[.,1]=sr[.,9];endif;
if jjj==2;rt4[.,2]=st[.,9];rr4[.,2]=sr[.,9];endif;
if jjj==3;rt4[.,3]=st[.,9];rr4[.,3]=sr[.,9];endif;
if jjj==4;rt4[.,4]=st[.,9];rr4[.,4]=sr[.,9];endif;
if jjj==5;rt4[.,5]=st[.,9];rr4[.,5]=sr[.,9];endif;
if jjj==6;rt4[.,6]=st[.,9];rr4[.,6]=sr[.,9];endif;
if jjj==7;rt4[.,7]=st[.,9];rr4[.,7]=sr[.,9];endif;

endo;                                    @ends loop in jjj that fills matrix@

format 6,0;"1990 " rt1;"2000 " rt2;"2010 " rt3;"2020 " rt4;
format 6,4;"1990 " rr1;"2000 " rr2;"2010 " rr3;"2020 " rr4;
                                                    @records matrix in file out@
endo;                        @ends loop in jjjj that does it for different f@
endo;            @ends jjjjj that makes different random year for the 3 inputs@
endo;             @ends jjjjjj that makes different random year for each cycle@
      @AAAAAA or 1 is same random year;BBBBBB or 2 is different random year@
endo;               @ends jjjjjjj, which is simple repetition of whole process@

vv=time-t0; hours=vv[1,.];min=vv[2,.];secs=vv[3,.];hun=vv[4,.];
ttime=hours*3600+min*60+secs+hun/100;
"minutes taken for simulation " ttime/60;
```

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