

PLP - A Package for Parametric Programming

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Golebiowski, A.

IIASA Working Paper

WP-88-118

December 1988

Golebiowski A (1988). PLP - A Package for Parametric Programming. IIASA Working Paper. IIASA, Laxenburg, Austria: WP-88-118 Copyright © 1988 by the author(s). http://pure.iiasa.ac.at/id/eprint/3088/

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WORKING PAPER

PLP - A PACKAGE FOR PARAMETRIC PROGRAMMING

A. Golebiowski

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS A-2361 Laxenburg, Austria

Foreword

This paper is one of the series of 11 Working Papers presenting the software for interactive decision support and software tools for developing decision support systems. These products constitute the outcome of the contracted study agreement between the System and Decision Sciences Program at IIASA and several Polish scientific institutions. The theoretical part of these results is presented in the IIASA Working Paper WP-88-071 entitled *Theory*, *Software and Testing Examples in Decision Support Systems* which contains the theoretical and methodological bacgrounds of the software systems developed within the project.

This paper presents the PLP package for parametric linear programming. This package constitutes the extension to MINOS, the well known linear and nonlinear programming code developed at Stanford University, and uses the MINOS as the solver of optimization problems. The PLP gives a complete parametric programming analysis for one, or more, of the following vectors: cost, rhs and bounds. In the same run several problems of this kind can be solved and for each, the starting point may be the original optimal solution obtained in the last problem. This property makes the PLP especially interesting for multiple criteria analysis using the reference point approach.

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PLP - A PACKAGE FOR PARAMETRIC PROGRAMMING

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INTRODUCTION

PLP is a software package for parametric linear programming. It is an extension of MINOS, the well-known linear and nonlinear programming code developed by Saunders and Murtagh^{*}. PLP is initiated by adding some specifications to the original list of MINOS specifications.

The package PLP uses MINOS as the solver of optimization problems. It includes sections which create an iterative framework for parametric programming and perform ranging and housekeeping procedures.

The formulation of the linear problem analyzed by PLP is similar as for MINOS.

Optionally, PLP gives a complete parametric programming analysis for one, or more, of the following vectors: cost, rhs and bounds. Of course such analysis can also be performed for single elements of these vectors. In the same run, several problems of this kind can be solved and for each, the starting point may be the original optimal solution or the final solution obtained in the last problem.

The last current complete solution in MINOS format is printed or stored with frequency specified by the user. Additionally, the user specifies the frequency of printing of a short message about current changes of optimal basis.

^{*} B.A. Murtagh and M.A. Saunders. MINOS - A Large-Scale Linear and Nonlinear Programming System. User's Guide. Technical Report Sol 77-9, Systems Optimization Laboratory, Stanford University California, 1977.

A. THEORETICAL GUIDE

1. GENERAL INFORMATION.

As options of PLP can be expressed in terms of the internal formulation of the linear problem used by MINOS we shall begin with recalling this concept.

The external formulation (supplied by the user) of the linear problem to be solved by MINOS is: Minimize (or maximize) a linear cost function

$$F(\mathbf{x}) = \mathbf{a}_0 \mathbf{x} \tag{1}$$

subject to m row constraints:

$$d_i \leq a_i x \leq g_i , \quad i = 1, \dots, m \tag{2}$$

and n constraints on separate variables:

$$d_{m+i} \leq x_i \leq g_{m+i}, \quad i = 1, ..., n$$
 (3)

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Here x is an *n*-dimensional column vector of decision variables, a_0 is an *n*-dimensional row vector of cost coefficients (also called the *objective row*), the a_i , i = 1,...,m, are ndimensional row vectors, the lower bounds d_i , i = 1, ..., m+n, are real numbers or $-\infty$, and the upper bounds g_i , i = 1, ..., m+n, are real numbers or $+\infty$. Of course, if the bounds take the values $+\infty$ or $-\infty$ the corresponding relation (2) or (3) must be replaced by a strict inequality. If $d_i = g_i$, then the variable x_i is said to be fixed. If $d_i = -\infty$ and $g_i = +\infty$ the variable x_i is said to be *free*. Analogous terms are used to describe the rows $a_i x$.

It should be recalled that in MINOS the two-sided inequality constraints (2) are not stated explicitly, but rather specified using ranges. More precisely, a one-sided inequality is introduced in the form $a_i x \leq g_i$ (type L) or $a_i x \geq d_i$ (type G), together with a real number r_i called the range. In the first case, the difference between the right-hand side g_i and this number yields the lower bound $(d_i = g_i - r_i)$; in the second case the sum of the right-hand side d_i and the real number r_i gives the upper bound $(g_i = d_i + r_i)$.

The linear programming problem is transformed by MINOS into the following internal form: Minimize (or maximize) the variable

$$-\tilde{x}_{n+1+\mathrm{obj}}$$
 (4)

subject to equality constraints:

$$\tilde{\mathbf{A}}\tilde{\mathbf{x}} = \mathbf{0} \tag{5}$$

and inequality constraints:

$$\tilde{l} \leq \tilde{x} \leq \tilde{u}$$
 . (6)

Here \tilde{A} is an $(m+1) \times (n+m+2)$ -matrix:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{1} & \tilde{b}_{1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \tilde{a}_{m+1} & \tilde{b}_{m+1} \end{bmatrix} , \qquad (7)$$

where I denotes the $(m+1) \times (m+1)$ identity matrix and

$$\tilde{a}_i = a_i \quad i < \text{obj} , \quad \tilde{a}_{\text{obj}} = a_0 , \quad \tilde{a}_i = a_{i-1} \quad i > \text{obj} ,$$

$$\tilde{b}_i = b_i \quad i < \text{obj} , \quad \tilde{b}_{\text{obj}} = 0 , \quad \tilde{b}_i = b_{i-1} \quad i > \text{obj} ,$$

$$(8)$$

where

$$b_i = \begin{cases} 0 & \text{if } d_i = -\infty \text{ and } g_i = +\infty \\ d_i & \text{if } d_i \text{ is finite and } g_i = +\infty \\ g_i & \text{if } g_i \text{ is finite} \end{cases}$$

The first *n* components of the extended vector of decision variables $\tilde{x} \in \mathbb{R}^{n+m+2}$ form a subvector identical to *x*; these components are described as *structural*. Element \tilde{x}_{n+1} is called the *right-hand-side component*; it is fixed at -1. The remaining components of \tilde{x} are called *slack* or *logical components*. The objective variable $\tilde{x}_{n+1+obj}$ is free. The vector of lower bounds \tilde{l} and the vector of upper bounds \tilde{u} are defined as follows:

$$\tilde{l}_{i} = d_{m+i} \quad i = 1, ..., n , \quad \tilde{l}_{n+1} = -1, \quad \tilde{l}_{n+1+obj} = -\infty ,$$

$$\tilde{u}_{i} = g_{m+i} \quad i = 1, ..., n , \quad \tilde{u}_{n+1} = -1, \quad \tilde{u}_{n+1+obj} = +\infty .$$
(9)

Now let i = n + 1 + j, j = 1,...,m. Then

$$\tilde{l}_i = h_i$$
, $\tilde{u}_i = k_i$ for $j < obj$ and $\tilde{l}_i = h_{i-1}$, $\tilde{u}_i = k_{i-1}$ for $j > obj$, (10)

where

 $h_i = k_i = 0$ if the *j*-th row constraint is fixed (i.e., of type *E*) (11) $h_i = 0, k_i = +\infty$ if $d_j = -\infty$ and g_j is finite (one-sided constraint of type *L*) $h_i = -\infty, k_i = 0$ if d_j is finite and $g_j = +\infty$ (one-sided constraint of type *G*) $h_i = 0, k_i = g_j - d_j$ if d_j and g_j are finite $h_i = -\infty, k_i = +\infty$ if the *j*-th row constraint is free.

2. MATHEMATICAL THEORY

This section presents elements of ranging theory for the linear programming problem (4)-(6). Some nonconventional notation will be used in order to avoid discussion of many particular cases. The sign \leq will denote "less than or equal to" if the expressions on its both sides are finite and "less than" otherwise. Similarly, \geq will denote "greater than or equal to" or "greater than". The notation $[t_1, t_2]$ will be used for the closure of the open interval (t_1, t_2) ; that is, t_1 and/or t_2 do not belong to the interval if they are not finite. For the sake of simplicity we shall assume that obj = m+1, i.e., the objective row is the last row in matrix \tilde{A} . As the value of variable \tilde{x}_{n+1} is fixed at -1 we may remove it from the problem formulation, defining a new column vector of decision variables $y \in \mathbb{R}^{n+m}$, where $y_i = \tilde{x}_i$ i = n+1, ..., n+m. We also define an m x (n+m)-matrix

$$A = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ a_m \end{bmatrix};$$

column vectors $b \in \mathbb{R}^m$ (see (8)), $l, u \in \mathbb{R}^{n+m}$, where $l_i = \tilde{l_i}$, $u_i = \tilde{u_i}$, i = 1,...,n and $l_i = h_{i+1}$, $u_i = k_{i+1}$, i = n+1,...,n+m; and a row vector $c \in \mathbb{R}_{n+m}$, where $c_i = a_0^i$

i = 1,...,n and $c^i = 0$ i = n+1,...,n+m.

The linear programming problem now takes the form: Minimize (or maximize) the linear cost function

$$F(\mathbf{y}) = c\mathbf{y} \tag{12}$$

subject to:

$$Ay = b \tag{13}$$

$$l \leq y \leq u \quad . \tag{14}$$

We denote the optimal solution of this problem by z and decompose it in the obvious way into the following subvectors:

 z_B - basic vector,

 z_l - vector of nonfixed, nonbasic variables which are at their lower bounds,

 z_u - vector of nonfixed, nonbasic variables which are at their upper bounds,

 z_i - vector of fixed variables (i.e., variables for which $u_i = l_i$).

Let I_u be the set of indices of all nonbasic variables at their upper bounds and let I_l be the set of indices of all nonbasic variables at their lower bounds. Fixed variables are not included in I_u or I_l . We shall let I_B denote the set of indices of all basic variables. This decomposition is also applied to the other vectors, yielding, for example, c_B , c_l , c_u ; l_B , l_l , l_u ; u_B , u_l , u_u . It is clear that $z_l = l_l$, $z_u = u_u$, $z_s = u_s$. Thus the constraint matrix is decomposed into the basic matrix B and matrices L, U, S such that

$$Bz_B + Lz_l + Uz_u + Sz_s = b$$

Hence we have

$$z_B = B^{-1}b - B^{-1}(Lz_l + Uz_u + Sz_s)$$
(15)

for the basic vector and

$$F(z) = c_B B^{-1} b + (c_l - c_B B^{-1} L) z_l + (c_u - c_B B^{-1} U) z_u + (c_s - c_B B^{-1} S) z_s \quad . \tag{16}$$

for the optimal cost.

Here and elsewhere we shall denote the *i*-th row of a matrix H by H_i and the *j*-th column by H^j . Define

$$D = B^{-1}$$
 . (17)

2.1. Parametric analysis of cost.

In every iteration of PLP COST the ranging problem has to be solved in the first place. Let Δc be a given nonzero row vector in R_{n+m} , where $\Delta c^i = 0$ for i = n+1,...,n+m and for fixed variables. We consider programming problems (12)-(13) with the cost vector c replaced by $\overline{c}(t)$, where

$$\overline{c}(t) = c + t\Delta c \tag{18}$$

and t is a real number, $t \in \mathbb{R}^1$. We wish to determine the largest range $[0, t_{\max}]$ in which the coefficient t may vary without affecting the optimal solution, i.e., the range of t values for which the optimal solution is equal to z.

It is clear from (16) that the optimal solution remains unchanged and equal to z for all values of t such that

$$\epsilon(\ \bar{c}_l(t) - \bar{c}_B(t)DL) \le 0 \tag{19}$$

and

$$\epsilon(\ \overline{c}_u(t) - \overline{c}_B(t)DU) \ge 0 \quad , \tag{20}$$

where

 $\epsilon = \begin{cases} +1 & \text{in the case of maximization} \\ -1 & \text{in the case of minimization} \end{cases}$

Hence

$$t\epsilon(\Delta c_l - \Delta c_B DL) \le \epsilon(c_B DL - c_l)$$

$$t\epsilon(\Delta c_u - \Delta c_B DU) \ge \epsilon(c_B DU - c_u) .$$
(21)

We shall use the following notation:

$$T_{j} = -c^{j} + c_{B}DA^{j}, \quad \Delta T_{j} = -\Delta c^{j} + \Delta c_{B}DA^{j}, \quad j \in I_{u} \cup I_{l} \quad .$$
⁽²²⁾

In the case of maximization we then have

$$t_{\max} = \min \left\{ -T_j / \Delta T_j \right\} \quad , \tag{23}$$

where the minimum is taken over all values of j from I_l such that $\Delta T_j < 0$ and all values of j from I_u such that $\Delta T_j > 0$.

In the case of minimization t_{\max} is determined from (23) but with the minimum taken over all values of j from I_i such that $\Delta T_j > 0$ and all values of j from I_u such that $\Delta T_j < 0$.

In all cases, if the set of indices over which the maximum (or minimum) is taken is empty, then $t_{max} = +\infty$.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for all $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of the optimal solution is determined by MINOS in the following way.

A shifted value of the cost vector is determined

$$c(t') = c + (t_{\max} + \Delta')\Delta c$$
⁽²⁴⁾

 Δ' is an appropriately chosen increment (see below). For this cost vector, MINOS finds the corresponding optimal solution. Next, the value of the cost vector and optimal cost at $t = t_{max}$ are calculated

$$\bar{c}(t_{\max}) = \bar{c}(t') - \Delta' \Delta c \tag{25}$$

$$F(t_{\max}) = \bar{c}(t_{\max})z \tag{26}$$

where z is the right-hand limit of the optimal solution for $t = t_{max}$ and Δ' is computed from:

$$\Delta' = DELTA * z \tag{27}$$

where DELTA is given by the user in the keyword PLP INCREMENT and x is the greatest real for which the following inequality is satisfied

$$-\boldsymbol{x} \Delta T_{\boldsymbol{i}} \leq \boldsymbol{f}(\boldsymbol{x}) \tag{28}$$

where

$$f(\mathbf{z}) = \begin{cases} TOLD^{*} || (\overline{c}_{B}t_{\max} + \Delta \overline{c}_{B}\mathbf{z})B^{-1} ||, || (\overline{c}_{B}t_{\max} + \Delta \overline{c}_{B}\mathbf{z})B^{-} || > 1 \\ TOLD , \text{ otherwise} \end{cases}$$
(29)

This inequality is solved for all values of the subscript *i* which belong to the set I_{\sum} (see

2.2. Parametric analysis of rhs.

In every iteration of PLP RHS the ranging problem has to be solved in the first place. Let Δb be a given nonzero column vector in \mathbb{R}^m . We consider the family of linear programming problems (12)-(14) with the rhs vector b replaced by $\overline{b}(t)$, where

$$\overline{b}(t) = b + t\Delta b \tag{30}$$

and $t \in \mathbb{R}^1$. We wish to determine the largest range $[0, t_{\max}]$ in which the coefficient t may vary without affecting the optimal basis, i.e., the range of t values for which the optimal basis is equal to B.

Letting $\bar{z}_B(t)$ denote the vector of basic variables in the optimal solution corresponding to the rhs vector $\bar{b}(t)$, we have

$$\overline{z}_B(t) = z_B + t B^{-1} \Delta b \quad . \tag{31}$$

It is clear that the nonbasic variables do not change for values of $t \in [0, t_{\max}]$. The range $[0, t_{\max}]$ is determined by the feasibility constraint on the basic variables:

$$l_B \le \bar{z}_B(t) \le u_B \tag{32}$$

or

$$l_B - z_B \leq tD\Delta b \leq u_B - z_B \quad . \tag{33}$$

Define

$$t_{1} = \min_{j=1,...,m} \left\{ \frac{u_{Bj} - z_{Bj}}{D_{j}\Delta b} : D_{j}\Delta b > 0 \right\}$$

$$t_{2} = \min_{j=1,...,m} \left\{ \frac{l_{Bj} - z_{Bj}}{D_{j}\Delta b} : D_{j}\Delta b < 0 \right\}$$
(34)

We then have

$$t_{\max} = \min\left\{t_1, t_2\right\} \tag{34}$$

If $D_i \Delta b \leq 0$ for all i, i = 1, ..., m, then we set $t_1 = +\infty$. Similarly, if $D_i \Delta b \geq 0$ for all i, i = 1, ..., m, then we set $t_2 = -\infty$.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of optimal solution is determined by MINOS in the following way.

A shifted rhs vector is determined

$$\bar{b}(t') = b + (t_{\max} + \Delta')\Delta b \tag{35}$$

 Δ' is an appropriately chosen increment (see below). For this rhs vector, MINOS finds the corresponding optimal solution. Next, the value of the rhs vector, the basic vector and the optimal cost at $t = t_{\text{max}}$ are calculated

$$\bar{b}(t_{\max}) = \bar{b}(t') - \Delta' \Delta b \tag{36}$$

$$z_B = B^{-1}\bar{b}(t_{\max}) - B^{-1}(Lz_l + Uz_u + Sz_s)$$
(37)

$$F(z) = c_B B^{-1} \bar{b}(t_{\max}) + (c_l - c_B B^{-1} L) z_l + (c_u - c_B B^{-1} U) z_u$$

$$+ (c_s - c_B B^{-1} S) z_s$$
(38)

where; z_l , z_u , z_s and z_B are the decomposition of the right hand side limit of the optimal solution for $t = t_{max}$. The matrices B, L, U, S are the decomposition of constraint matrix A valid for the optimal solution for $t = t_{max}$.

 Δ ' is computed from

$$\Delta' = DELTA * x \tag{39}$$

where DELTA is given by the user in the keyword PLP INCREMENT and x is the greatest real for which the following inequality is satisfied

$$x \mid D_{i\max} \Delta b \mid \leq TOLX \tag{40}$$

where j_{max} is the subscript for which t_{max} is calculated in formula (34).

2.3. Ranging of bounds.

In every iteration of PLP BOUND the ranging problem has to be solved in the first place. Let col $(\Delta l, \Delta u)$ be a given column vector in $R^{2(n+m)}$, and be such that $\Delta l_i = \Delta u_i = 0$ if y_i is a fixed variable. We consider the family of linear programming problems (A.1) - (A.3) with the vectors of lower and upper bounds l and u replaced by $\overline{l}(t)$ and $\overline{u}(t)$, respectively, where

$$\bar{l}(t) = l + t\Delta l, \quad \bar{u}(t) = u + t\Delta u \tag{41}$$

and $t \in \mathbb{R}^1$. We wish to determine two ranges, $[0, t_{maxa}]$ and $[0, t_{maxb}]$. The first of these intervals is the largest range in which the coefficient t may vary without affecting the optimal solution (i.e., the range of t values for which the optimal solution remains equal to z); the second is the largest range in which t may vary without affecting the optimal basis (i.e., the range of t values for which the optimal basis remains equal to B).

The boundary t_{maxa} is easily determined from the following conditions: for every $t \in [0, t_{maxa}]$

$$t\Delta l_{i}=0 \text{ if } i \in I_{l}$$

$$t\Delta u_{i}=0 \text{ if } i \in I_{u}$$

$$l_{i}+t\Delta l_{i} \leq u_{i} \text{ if } i \in I_{u}$$

$$u_{i}+t\Delta u_{i} \geq l_{i} \text{ if } i \in I_{l}$$

$$l_{i}+t\Delta l_{i} \leq z_{i} \leq u_{i}+t\Delta u_{i} \text{ if } i \in I_{B}.$$

$$(42)$$

The first two conditions imply that $t_{\max} = 0$ if $\Delta l_i = 0$ for some $i \in I_l$ and/or $\Delta u_i = 0$ for some $i \in I_u$.

Let $\overline{z}(t)=z+t\Delta z$ denote the optimal solution corresponding to the vector of bounds col $(\overline{l}(t),\overline{u}(t))$. Then

$$\Delta z_l = \Delta l_l , \ \Delta z_u = \Delta u_u$$

$$\Delta z_B = -D(L\Delta l_l + U\Delta u_u)$$
(43)

The values of t_{maxb} may be calculated using the feasibility conditions

$$l_{l}+t\Delta l_{l} \leq u_{l}+t\Delta u_{l} , \ l_{u}+t\Delta l_{u} \leq u_{u}+t\Delta u_{u}$$

$$l_{B}+t\Delta l_{B} \leq z_{B}+t\Delta z_{B} \leq u_{B}+t\Delta u_{B}$$

$$(44)$$

or

$$t(\Delta l_l - \Delta u_l) \le u_l - l_l$$

$$t(\Delta l_u - \Delta u_u) \le u_u - l_u$$
(45)

$$t(\Delta l_B + DL\Delta l_l + DU\Delta u_U) \le z_B - l_B$$
$$t(\Delta u_B + DL\Delta l_l + DU\Delta u_u) \ge z_B - U_B$$

Define

$$t_{1} = \min_{j \in B} \left\{ \frac{u_{j} - l_{j}}{\Delta l_{j} - \Delta u_{j}} : \Delta l_{j} - \Delta u_{j} > 0 \right\}$$

$$t_{2} = \min_{j=1,...,m} \left\{ \frac{z_{B_{j}} - l_{B_{j}}}{\Delta u_{B_{j}} + D_{j}(L\Delta l_{l} + U\Delta u_{u})} : \text{denominator} < 0 \right\}$$

$$t_{3} = \min_{j=1,...,m} \left\{ \frac{z_{B_{j}} - l_{B_{j}}}{\Delta l_{B_{j}} + D_{j}(L\Delta l_{l} + U\Delta u_{u})} : \text{denominator} > 0 \right\}$$

$$(46)$$

Finally,

$$t_{\text{maxb}} = \min\{t_1, t_2, t_3\}$$
 (47)

If the set of indices j over which a minimum is taken is empty, we substitute $+\infty$ for t_1 , t_2 , or t_3 in (46). For instance, if $\Delta l_j - \Delta u_j \leq 0$ for all $j \in B$, we take $t_1 = +\infty$, and so on.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of optimal solution is determined by MINOS in the following way.

The shifted vectors of lower and upper bounds are determined

$$\overline{l}(t') = l + (t_{\text{maxb}} + \Delta')\Delta l$$

$$\overline{u}(t') = u + (t_{\text{submaxb}} + \Delta')\Delta u$$
(48)

where Δ' is an appropriately chosen increment (see below). For these bound vectors, MINOS finds the corresponding optimal solution. Next, the values of the bound vectors, the basic vector and the optimal cost at t = tmaxb are calculated

$$\overline{l}(t_{\max b}) = \overline{l}(t') - \Delta' \Delta l \tag{49}$$
$$\overline{u}(t_{\max b}) = \overline{u}(t') - \Delta' \Delta u$$

$$\begin{aligned} u(t_{\text{maxb}}) &= u(t) - \Delta \Delta u \\ z_B &= B^{-1}b - B^{-1}(Lz_l(t_{\text{maxb}}) + Uz_u(t_{\text{maxb}}) + Sz_s) \end{aligned}$$
(50)

$$F(z) = c_B B^{-1} b + (c_l - c_B B^{-1} L) z_l(t_{\text{maxb}}) + (c_u - c_B B^{-1} U) z_u(t_{\text{maxb}})$$
(51)
+ $(c_s - c_B B^{-1} S) z_s$

where z_l , z_u , z_s and z_B are the decomposition of the right-hand side limit of the optimal solution for $t = t_{maxb}$. The matrices B, L, U, S are the decomposition of the constraint matrix A valid for the optimal solution $t = t_{maxb}$ and Δ' is computed from:

$$\Delta' = DELTA * x \tag{52}$$

where DELTA is given by the user in the keyword PLP INCREMENT and

$$\boldsymbol{x} = \frac{TOLX}{|f|} \tag{53}$$

f is the denominator of that fraction in the two last definitions (46) which is equal to t_{maxb} .

3. THE METHODS.

3.1. The method of PLP COST.

The algorithm of PLP COST is as follows:

- 1. Set i := 0, $t_i := 0$.
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and the basic vector z_B .
- 3. The boundary value of the parameter t_{i+1} is calculated, such that for all $a_0(t), t \in [t_i, t_{i+1})$ the optimal solution is constant. The set I_{∞} of nonbasic variables is determined, containing all nonbasic variables for which reduced costs:

$$a_0^{k}(t) - \tilde{a}_B(t)B^{-1}\tilde{A}^k \tag{54}$$

where \tilde{A}_k is the k-th column of the constant matrix \tilde{A} (see (7)), reach zero for some t in the interval $[t_i, t_i + 10^{-9})$. These variables are nonbasic in the decomposition valid for $t = t_i$.

4. Next, the value t' of the parameter is determined:

$$t' = t_{i+1} + \Delta', \, \Delta' = DELTA *\Delta, \, DELTA > 1$$
(55)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the nonbasic variable whose reduced cost reaches zero at t_{i+1} is still recognized by MINOS as optimal.

5. New cost vector is computed:

$$a_0(t') = a_0(t_i) + (t' - t_i)\Delta a_0$$
(56)

- 6. MINOS finds the new optimal solution for the new cost vector $a_0(t')$.
- 7. Set $t_i := t_{i+1}$ and shift the cost vector back to t_i

$$a_0(t_i) := a_0(t') - \Delta' \Delta a_0 \tag{57}$$

8. Set i := i+1 and go to 3.

3.2. The method of PLP RHS.

The algorithm of PLP RHS is as follows:

- 1. Set i := 0 and $t_i := 0$.
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and basic vector z_B . At the same time it finds the optimal decomposition into basic and nonbasic variables.
- 3. The boundary value of the parameter t_{i+1} is calculated (see section 2.2), such that for all $\tilde{b}(t)$, $t \in [t_i, t_{i+1})$ the optimal basis (basic matrix) is constant and equal to B. The set I_{\sum} of the basic variables is determined containing all basic variables which reach their bounds for some value of t in the interval $[t_{i+1}, t_{i+1} + 10^{-9}]$. These variables are basic in the decomposition valid for $t = t_i$.
- 4. Next, the value t' of the parameter is determined

$$t' = t_{i+1} + \Delta', \ \Delta' = DELTA^*\Delta, DELTA > 1$$
(58)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the basic variable which reaches its bound at t_{i+1} is still recognized by MINOS as feasible.

5. New rhs vector is computed

$$\tilde{b}(t') = \tilde{b}(t_i) + (t' - t_i) \Delta \tilde{b}$$
(59)

and the corresponding starting basic solution

$$z_B(t') = z_B(t_i) + (t' - t_i)B^{-1}\Delta \vec{b}$$
(60)

- 6. MINOS finds the optimal solution for the new rhs vector $\tilde{b}(t')$, starting from the shifted basic solution (60) which is infeasible. The new optimal basis is denoted by B and the new basic vector by $z_B(t')$
- 7. Set $t_i := t_{i+1}$ and shift the solution back to t_i ,

$$z_B(t_i) = z_B(t') - (t' - t_i)B^{-1}\Delta\tilde{b}$$
(61)

also

$$\tilde{b}(t_i) = b(t') - (t' - t_i) \Delta \tilde{b}$$
(62)

8. Set i := i + 1 and go to (3).

3.3. The method of PLP BOUND

The algorithm of PLP BOUND is as follows:

- 1. Set i := 0 and $t_i := 0$
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and the basic vector z_B . At the same time it finds the optimal decomposition into the basic and nonbasic variables.
- 3. The boundary value of the parameter t_{i+1} is calculated (see section 2.3), such that for all $\tilde{l}(t)$ and $\tilde{u}(t)$, $t \in [t_i, t_{i+1})$ the optimal basis (basic matrix) is constant and equal to *B*. The set I_{\sum} of basic variables is determined, containing all basic variables which reach their bounds for some value of *t* in the interval $[t_{i+1}, t_{i+1} + 10^{-9}]$. These variables are basic in the decomposition valid for $t = t_i$.
- 4. Next, the value t' of the parameter is determined

$$t' = t_{i+1} + \Delta'$$
, $\Delta' = DELTA^*\Delta$, $DELTA > 1$ (63)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the basic variable which reaches its bound at t_{i+1} is still recognized by MINOS as feasible.

5. New bound vectors are computed:

$$\tilde{l}(t') = \tilde{l}(t_i) + (t' - t_i)\Delta \tilde{l}$$

$$\tilde{u}(t') = \tilde{u}(t_i) + (t' - t_i)\Delta \tilde{u}$$
(64)

and the corresponding starting basic solution:

$$z_B(t') = z_B(t_i) + (t' - t_i)B^{-1}(L\Delta \tilde{l} + U\Delta \tilde{u}).$$
(65)

MINOS finds the optimal solution for the new bound vectors, starting from the shifted basic vector (65) (which is infeasible). The optimal basis is denoted by B and the basic vector by $z_B(t')$.

7. Set $t_i := t_{i+1}$ and shift the solution back to t_i

$$z_B(t_i) = z_B(t') - (t'-t)B^{-1}(L\Delta \tilde{l} + U\Delta \tilde{u}).$$
(66)

- 13 -
- 8. Set i := i+1 and go to 3.

B. USER MANUAL

1. BRIEF CHARACTERIZATION OF BASIC FUNCTIONS OF PLP.

1.1. Parametric analysis of cost (PLP COST).

The cost vector $a_0 = (a_0^1, a_0^2, \dots, a_0^n)$ (see (1)) is changed along a direction given by the user, $\Delta a_0 = (\Delta a_0^1, \Delta a_0^2, \dots, \Delta a_0^n)$ according to the formula:

$$a_0(t) = a_0(0) + t\Delta a_0, \ t \ge 0 \tag{67}$$

where $a_o(0)$ is the starting value of cost. If the structural variable, say \tilde{x}_i , is fixed then Δa_0^{i} is automatically set to zero, regardless of the value given in the data.

PLP determines a sequence of values of the parameter denoted by t_0, t_1, \ldots, t_k , such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal solution is constant and in each case the optimal basis is different. The integer k: (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal solution is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k)$, (3) may be determined by the condition that there are no optimal solutions for every $t > t_k$.

1.2. Parametric analysis of rhs (PLP RHS).

The right-hand side $\tilde{b} = \operatorname{col}(\tilde{b_1}, \ldots, \tilde{b_{m+1}})$ $\tilde{b} = \operatorname{col}(\tilde{b_1}, \ldots, \tilde{b_{m+1}})$ (see (7) and (8)) is changed along a direction given by the user, $\Delta \tilde{b} = \operatorname{col}(\Delta \tilde{b_1}, \ldots, \Delta \tilde{b_{m+1}})$, according to the formula:

$$\tilde{b}(t) = \tilde{b}(0) + t\Delta \tilde{b} , t \ge 0$$
(68)

where $\tilde{b}(0)$ is the starting value of rhs. The component of $\Delta \tilde{b}$ which corresponds to the objective row is automatically set to zero, $\Delta \tilde{b}_{obj} = 0$.

PLP determines a sequence of values of the parameter denoted by t_0, t_1, \ldots, t_k such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal basis is constant and in each case different. The integer k : (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal basis is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k), (3)$ may be determined by the condition that there are no feasible solutions for every $t > t_k$.

1.3. Parametric analysis of bounds (PLP BOUND).

The vector of bounds $\operatorname{col}(\tilde{l}, \tilde{u}) \in R^{2(n+m+2)}$ (see (9)) is changed along a direction given by the user, $\operatorname{col}(\Delta \tilde{l}, \Delta \tilde{u})$, according to the formula:

$$\operatorname{col}(\tilde{l}(t), \, \tilde{u}(t)) = \operatorname{col}(\tilde{l}(0), \, \tilde{u}(0)) + t \, \operatorname{col}(\Delta \tilde{l}, \, \Delta \tilde{u}) \, , \, t \ge 0 \tag{69}$$

where $\operatorname{col}(\tilde{l}(0), \tilde{u}(t))$ is the starting value of bounds. The bound increments $\Delta \tilde{l}_i \Delta \tilde{u}_i$ which correspond to fixed variables are automatically set to zero regardless of the values given in the data.

If there is no lower and/or upper bound for the i-th variable \tilde{x}_i (see (6)) the corresponding increment $\Delta \tilde{l}_i$ and/or $\Delta \tilde{u}_i$, respectively, is also automatically set to zero.

PLP determines a sequence of values of the parameter denoted by t_0, t_1, \ldots, t_k such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal basis is constant and in each case different. The integer k : (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal basis is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k)$, (3) may be determined by the condition that there are no feasible solutions for every $t > t_k$.

Each interval $[t_i, t_{i+1}]$ is optionally divided into two subintervals $[t_i, t_i^a], [t_i^a, t_{i+1}]$. The interval $[t_i, t_i^a]$ is the maximum interval where the optimal solution remains constant and not only the optimal basis. It often happens that $t_i = t_i^a$.

1.4. Dependent and independent work.

All three kinds of analysis can be performed in one run. The starting point for the next kind of analysis may be either the original starting optimal solution (The Independent Work) or the last optimal solution obtained in the preceding analysis (The Dependent Work). The continuation is impossible if the optimal solution vanishes.

1.5. Controlling output.

In each of the three kinds of analysis the following information is available. The user has to specify the frequency of printing the complete current optimal solution in MINOS format. This means that the complete printout is given for the values of parameters tequal to t_{0+} , t_{p+} , t_{2p+} ,..., and $t_{(k-1)+}$ or t_{k+} depending on whether the optimal solution exists for $t > t_k$. The notation t_{i+} means that we take the right-hand limit of the optimal solution at t_i . The user specifies frequency of printing the so called PLP ITERATION LOG. This is a short message containing most important information about current change of optimal solution (value of the parameter t, change of optimal basis, current value of objective function).

1.6. Tolerances.

The performance of PLP is strongly affected by the choice of tolerances. Especially important are two tolerances determined in MINOS : the tolerance of optimality (TOLD) and the tolerance of feasibility (TOLX). In the proper procedures of the PLP the following general rule is adopted. All quantities greater than or equal to 1.E+15 are taken as equal to infinity and all quantities whose absolute value is less than 1.E-9 are regarded as equal to zero.

2. INPUT

The input contains all necessary elements for MINOS with the conditions given below.

Key		Default	Meaning		
PLP ANALY	COST 'SIS n	off	This keyword activates the parametric analysis of cost. The integer n is the number of iterations to be performed. If no value or a zero value of n is given, all iterations will be performed (until the optimal solution becomes constant or the optimal solution vanishes).		
PLP ANALY	RHS SIS n	off	This keyword activates the parametric analysis of rhs. The integer n is the number of iterations to be performed. If no value or a zero value of n is given, all iterations will be performed (until the optimal basis becomes constant or the optimal solution van- ishes).		

2.1. I	New]	key-word	s in the	SPECS	file
--------	-------	----------	----------	-------	------

PLP BOUND ANALYSIS n	off	This keyword activates the parametric analysis of bounds. The absolute value of integer n is the number of iterations to be performed. If n is less than zero an additional output is printed in each iteration which gives the values of $t_i^a - t_i$ and $t_{i+1} - t_i$ and the corresponding boundary values of bounds.
PLP ORDER	off	This keyword activates the dependent work of PLP. If it does not occur, PLP performs each of the required kinds of analysis only once (keywords PLP COST, PLP RHS, PLP BOUND). In each of these, the starting point is the original optimal solu- tion. If PLP ORDER appears in the SPECS FILE, it must precede the sequence of keywords PLP COST, PLP RHS, PLP BOUND, which define the kinds of analysis to be performed in the same order. For each kind of analysis, the starting point is the last optimal solution obtained in the last analysis. If the optimal solution vanishes, the run stops. Each kind of analysis can be performed up to five times, in an arbitrary order (determined by the sequence of keywords PLP COST, PLP RHS, PLP BOUND). In each repetition of the same kind of analysis, the search direction and the max- imum number of iterations must be the same. The value of n given in the last keyword referring to a particular kind of analysis is valid for all its repeti- tions.
PLP SOLN n	n = 1	This keyword specifies the frequency of printing the current complete solution in the MINOS format. Full solution is printed after every n iterations. If this keyword is omitted or $n = 0$, the effect is the same as for $n = 1$.
PLP FREQUENCY n	n = 1	This command activates the frequency of printing the short message called PLP ITERATION LOG (see section 3 of USER MANUAL). A PLP ITERA- TION LOG is printed after every n iterations. If this keyword is omitted or $n = 0$, the effect is the same as for $n = 1$.
PLP SOLU- TION n	off	If this (optional) keyword is used with $n \ge 0$ complete outputs of optimal solution will be stored in file n with the frequency given in PLP SOLN m . If $n=0$ or this keyword does not occur, the complete outputs are stored in the printer file.
PLP FILE n	n = 5	The absolute value of n is the logical number of the data file for parametric programming. This file is read after processing other MINOS files has been completed. The parameter n also controls the output of the search directions. If n is less than zero, the search direction of each PLP analysis is printed. These directions are not printed for any other entry.

PRINT DATA PLP FILE	off	If this keyword is used, the whole DATA PLP FILE will be printed in the output. Otherwise, only the records with comments and the records NAME, SET and ENDATA are printed.
PLP INCREMENT d	d = 1.1	This keyword specifies the value of factor Δ (see sections 3.1, 3.2 and 3.3 of THEORETICAL GUIDE). (4.2), (4.3)).

2.2. DATA PLP file - input format.

The data for the PLP procedures are prepared in an MPS-like format and placed in the file specified by the key-word DATA PLP FILE n. The data sets for different PLP procedures may be given in any order. The beginning of the data set for each procedure is identified by the line NAME and its end by the line ENDATA. If it occurs, the line 'SET' must be given immediately after the line NAME in each data set; this line defines the default values of all the variables which are not explicitly defined. Every data set is identified by the name given in the line NAME.

The records in the DATA RANGING FILE should have the following (basic) form, which is analogous to MPS format:

Columns:	1-4,	5-12,	15-22,	25-36,	40-47,	5061
Fields:	f1,	f2,	f3,	f4,	f5,	f6

Below we give a detailed description of the data set for each parametric programming procedure.

Parametric analysis of cost (PLP COST)

	f1	f2	f3	f4	f5	f6
1. 2. 3. 4.	NAME 'SET' ENDATA	С	PLPC Comments Col. name	Value Value	Col. name	Value

Parametric analysis of rhs (PLP RHS)

	f1	f2	f3	f4	f5	f6
1. 2.	NAME 'SET'		PLPR Comments	Value		
3.	011		Row name	Value	Row name	Value
4.	ENDATA					

Parametric analysis of bounds (PLP BOUNDS)

	f1	f2	f3	f4	f5	f6
1. 2. 3. 4. 5.	NAME 'SET' ENDATA		PLPB omments Row/col. name Row/col. name		Row/col. name Row/col. name	Value Value

Remarks:

- if field f2 in a given record is empty, this means that it is the same as in the previous record. Field f2 must not be empty in the first data record,
- the records with identifiers UPPER and LOWER may appear in any order,
- LOWER is used for increments of the lower bounds and UPPER for increments of the upper bounds.

The following general rules apply to all data sets:

- One of the fields f3, f5, (f4, f6) may be empty.
- If 'SET' appears, it must follow immediately after NAME. If 'SET' does not occur, the default for all variables whose values are not specified is zero. This has the same effect as:

'SET' 0.

- Comments may be entered in arbitrary positions in the data set. They are identified by an asterisk * in the first column.
- The values should be written as real numbers in a format accepted by FORTRAN.

2.3. Specification of zeros in the MPS file.

In two kinds of parametric analysis, PLP COST and PLP RHS the user has to specify explicitly some of the zero values of the objective row elements (vector a_0) and/or the rhs column elements (vector \tilde{b}), exactly in the same way as the nonzero values specified in the data (MPS file). This refers to those elements of the vector a_0 and/or \tilde{b} for which the corresponding elements of Δa_0 and/or $\Delta \tilde{b}$, respectively, are different from zero.

Example $a_0 = (1.,0.,0.,3.,5.)$, $\Delta a_0 = (-1.,0.,0.1,1.,0.)$ In this case the element a_0^3 has to be explicitly specified in MPS

x3 obj 0.

where x_3 is the name of the third column (structural variable) and *obj* is the name of the objective row.

3. OUTPUT

The title of the output of PLP is:

P L P VERSION 1.0 JUNE 1986

In the case of dependent work of PLP the subtitle is printed:

DEPENDENT WORK OF PLP

Otherwise, this subtitle is omitted.

The output may be sent either to the printer file or to the file defined by the keyword PLP SOLUTION FILE. Only the output produced by the procedure SOLN of MINOS can be stored in the latter one.

Since the SOLN output is described in MINOS manuals we will confine ourselves to the output of PLP sent to the printer file, and so we will also skip the messages given by MINOS.

Each kind of parametric analysis procedures produces a printout containing the following information.

Title:

PLP COST	- for parametric analysis of cost
PLP RHS	- for parametric analysis of rhs
PLP BOUND	- for parametric analysis of bounds

Search direction (optionally):

For PLP COST it has the following format. For each structural variable \tilde{x}_i , i = 1, ..., n the following information is given:

- NUMBER Number of structural variable
- COLUMN Name of structural variable

DIRECTION - Increment component Δa_0^i

OBJ GRADIENT- Cost component a_0^i

 $\mathbf{M}+\mathbf{J} \qquad -m+1+i$

In the case of PLP RHS the following information is given for each row (or each slack variable \tilde{x}_i , i = n+2, ..., n+m+2) except for the objective row (or slack variable \tilde{x}_{n+1} +obj):

NUMBER	- Number of slack variable
ROW	- Name of row
DIRECTION	- Component $\Delta \tilde{b_i}$ of increment vector
RHS	- Right-hand-side component $ ilde{b_i}$
Ι	- Row number

For PLP BOUND this printout is divided into two sections:

SECTION 1 - ROWS contains the following information for each slack variable \tilde{x}_i , $i = n+2, \ldots, n+m+2$ (or for each row), except for the slack variable \tilde{x}_{n+1+} obj which corresponds to the objective row:

NUMBER - Number of slack variable

ROW - Name of row

LL DIRECTION- Component $\Delta \tilde{l}_i$ of the lower bound inc. vector $\Delta \tilde{l}$

LOWER LIMIT- Lower bound $\tilde{l_i}$

UL DIRECTION- Component $\Delta \tilde{u}_i$ of the upper bound inc. vector $\Delta \tilde{u}$

UPPER LIMIT- Upper bound \tilde{u}_i

I - Row number

SECTION 2 - COLUMNS contains information analogous to that described above for each structural variable \tilde{x}_i , i = 1, ..., n with the following differences:

NUMBER - Number of structural variable

COLUMN - Name of structural variable

M+J -m+1+i

PLP iteration log printing:

Printing frequency is given in the keyword PLP FREQUENCY. It takes one of the following forms:

If only one variable in the optimal basis has been exchanged and none of the nonbasics has changed its state, the following message is printed:

PITN - Number of iteration of current parametric analysis

OBJ - Objective value

TMAX - Current boundary value of parameter t

VARIABLE "name" (number of the variable) FROM "bound" REPLACES BASIC VARI-ABLE "name" (number of the variable) WHICH PASSES TO "bound"

(LL is substituted for "lower bound" and UL for "upper bound")

In other cases the first three items are the same as above and the last row is replaced by the appropriate number of the following sentences:

VARIABLE"name" (number of the variable) FROM "bound" ENTERS THE BASIS

BASIC VARIABLE"name" (number of the variable) PASSES TO "bound"

VARIABLE"name" (number of the variable) FROM "bound" PASSES TO "bound"

If a variable which does not belong to I_{\sum} has changed its state, this row is preceded by the following message:

WITHIN THE GIVEN TOLERANCE ONLY THE FOLLOWING INFORMATION IS AVAILABLE

Special messages

1. If in the final iteration the situation arises in which the optimal basis is constant for every $t > t_{max}$, the following message appears in the printer file:

PITN - Number of iteration of current parametric analysis - FOR THE VALUE OF THE PARAMETER = Value of t_i INFINITE RANGE (TMAX.GE.1.E15)

where $\text{TMAX} = t_{i+1} - t_i$. In this case the last optimal solution is stored in the printer file or in the file defined by the keyword PLP SOLUTION FILE.

- 2. If the optimal solution vanishes, one of the following MINOS messages is printed:
 - in the case of PLP COST:

EXIT - PROBLEM IS UNBOUNDED

this is followed by:

PITN - Number of iteration of current parametric analysis TMAX= Boundary value of the parameter t

- for PLP RHS and PLP BOUND:

EXIT - PROBLEM IS INFEASIBLE NO. AND SUM OF INFEASIBILITIES "number" and "value"

This is followed by:

PITN = Number of iteration of current kind of analysis TMAX = Boundary value of parameter

In both cases the SOLN output corresponding to the value t_{i+1} of parameter t is printed or stored in the file defined by the user in the keyword PLP SOLUTION FILE.

3. If MINOS cannot find the next optimal solution because of tolerances defined in MINOS, the following printout is displayed:

WITHIN THE GIVEN TOLERANCE NO NEW BASIS IS FOUND

This is a failure of the package. In order to continue the analysis, the user should decrease the appropriate tolerance (tolerances) in MINOS or to increase the factor DELTA in keyword PLP INCREMENT.

4. If the keyword PLP BOUND ANALYSIS n is less than zero an additional output is printed. It gives the values: $t_i^a - t_i = t_{maxa}$, $t_{i+1} - t_i = t_{maxb}$ and the corresponding boundary values of bounds:

PITN = Number of iteration of current kind of analysis TMAX = boundary value of parameter

This is followed by the information on t_{maxa} .

4. EXAMPLES

We shall now illustrate the performance of PLP using a simple example. The linear programming problem is as follows :

Maximize

 $F(x)=0.1x_1+x_2$

subject to:

 $x_1 + x_3 = 3.$ 0.7065 $(x_1 + x_2) + x_4 = 3.826$

$$\begin{aligned} x_2 + x_5 &= 3. \\ -0.7065(x_1 - x_2) + x_6 &= 1. \\ -x_1 + x_7 &= -1 \\ 0 &\leq x_1 , x_2 &\leq 5 , 0 \leq x_i \leq 2 , i = 3, \dots, 7. \end{aligned}$$

Two runs of PLP are presented. The first shows the independent work of PLP. It contains all three kinds of parametric analysis: PLP COST, PLP RHS, PLP BOUND. In the second, we have the results of dependent work of PLP. The task for PLP was to perform one iteration of PLP RHS, then all iterations of PLP COST and then all iterations of PLP BOUND.

Below we give the MPS file common for both runs and then we give the MINOS and PLP specifications used to solve each of these problems.

Then we give the standard MINOS printout, followed by two outputs of PLP.

2 Martine and a Martine and	test	
TOWE		
n ob		
e ri		
• r 2		
• r3		
• r 4		
• r5		
columns		
x1	ob	0.1
x1	r1	1.
x1	r2	.7065
x1	r 4	7065
x1	r6	-1.
x2	ob	1.
x2	r2	.7065
x2	r 3	1.
x2	r 4	. 7065
x3	ri	1.
x4	r2	1.
305	r3	i .
305	r 4	1.
x 7	r6	1.
rhs		
rh	ri	3.
rh	r2	3.826
rh	r3	3.
rh	r 4	1.
rh	r5	-1.
bounds		
zzi bo	x2	
up bo	x2	Б.
up bo	хЗ	2.
up bo	204	2.
up bo	x6	2.
up bo	365	2.
up bo	x7	2.
endata		

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specs file

-		
	begin	Second P L P Test
		maximize
		plp rhs analysis
		plp cost analysie
		plp bound analysis -10
		plp increment 1.5
		plp frequency 1
		data plp file -9
		print data plp file

end

pro	ðlæn	name	test	c	bjective value	3.24154282154+00				
sta	tus		optimal	.soln i	iteration 1	superbasics	0			
obj	ectiv	•	оЪ	(max)						
rhe			rh							
ran	ges									
bou	nds		Ъо							
500	tion	1 - rom	18							
נות	mber	ro	r at	activity	<pre>slack activity</pre>	lower limit.	upper limit.	dual activity	i	
	9	ob	bs	3.24154	-3.24154	none	none	1.00000	1	
a	10	r1	eq	3.00000	.00000	3.00000	3.00000	.00000	2	
	11	r 2	eq	3.82600	.00000	3.82600	3.82600	14154	3	
	12	r 3	eq	3.00000	.00000	3.00000	3.00000	90000	4	
a	13	r 4	eq.	1.00000	.00000	1.00000	1.00000	.00000	5	
8	14	r 5	eq	-1.00000	.00000	-1.00000	-1.00000	. 00000	6	
500	tion	2 - co	lumis							
טמ	mber	.colur	m. at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m+j	
	1	x1	bs	2.41543	. 10000	.00000	none	. 00000	7	
	2	x2	bs	3.00000	1.00000	none	5.00000	, 00000	8	
	3	хЗ	bs	.58457	. 00000	.00000	2.00000	. 00000	9	
	4	xx4	ш	.00000	. 00000	. 00000	2.00000	14154	10	
	5	хб	ш	.00000	. 00000	.00000	2.00000	90000	11	
	6	x6	bs	.58700	.00000	.00000	2.00000	. 00000	12	
	-	-								

.00000

.00000

.00000

-1.00000

2.00000

-1.00000

.00000

-3.24154

13

14

PLP --- varsion 1.0 june 1986, = = =

1.41543

-1.00000

data plp file

7 x7

8 rh

bs

eq

1	nao		plpc		
2	180	rt'		.00000001+00	
3			x1	1.0000001+00	00+500000.
4			x2	-1.00000d+00	.000004+00
Б	*				
6	*	Note:			
7	*	Declarat:	ion of du	mmy coefficients (=0 i	in MPS file) of the objective
8	*	is not no	ecessary	because the above dire	ection is defined in the x1-x2
9	*	subspace	of cost	vectors.	
10	*				

11 endata

plp c	ost
-------	-----

numbe		alumma diin	rectionobjg	radient. m	+i			
					-			
	1 x1		1.00000	.10000	7			
	2 x2		-1.00000	1.00000	8			
	3 x3		.00000	.00000	9			
	4 x4 5 x5		.00000 .00000		10 11			
	8 x6		.00000		12			
	7 x7		.00000		13			
pitn=	1	obj= 0.2	297848564+01	tamesc= 0	.450004+00	variable 35	(5) from 11	
replac	es bas	ic variable	x3 (3)w	hich passes	to 11			
proble	n nami	test	obj∉	ctive value	2.97848556321+	-00		
status		optimal so	oln iter	ration 1	superbasics	0		
object	ive	ob	(mex)					
rhs		rh						
ranges								
bounds		ро						
sectio	n 1 -	TOWS						
numbe	r	.row. at	activity sl	lack activity	lower limit.	upper limit.	dual activity	i
	9 ob	bs	2.97849	-2.97849	none	none	1.00000	1
1	0 r1	eq	3.00000	. 00000	3.00000	3.00000	0.00000	:
1	1 r2	eq	3.82600	. 00000	3.82600	3.82600	77848	3
1	2 r3	eq	3.00000	. 00000	3.00000	3.00000	0.00000	4
	3 r 4	ed	1.00000	. 00000		1.00000	. 00000	Ę
a 1	4 175	eq	-1.00000	. 00000	-1.00000	-1.00000	. 00000	e
sectio	n 2 -	columns						
numbe	er .co	olumn. at	activityc	obj gradient.	lower limit.	upper limit.	reduced gradnt	m +
	1 x1	bs	3.00000	. 55000	. 00000	none	. 00000	7
	2 x2	bs	2.41543	. 55000	none	Б.00000	. 00000	8
	3 x3	11	.00000	.00000	.00000	2.00000	0.00000	S
	4 x4	11	.00000	.00000	.00000	2.00000	77848	10
	5 x55	bs	.58457	. 00000	.00000	2.00000	0.00000	11
	6 x6	bs	1.41300	. 00000		2.00000	.00000	12
	7 x7	bs	2.00000	.00000		2.00000	.00000	13
	8 rh	eq	-1.00000	. 00000	-1.00000	-1.00000	-2.97849	14
pitn=		-	329999501+01		.10000d+01	variable x4	(4) from 11	
replac	186 DE.	sic variable	200 (0)1	nhich passes	CO 81			
proble		n test	obje	ctive value	3.29999500964	+00		
status	I	optimal so	oln iter	ration 2	superbasics	0		
object	ive	ob	(max)					
rhs		rh						
ranges	I							
bounds		Ъо						

section 1 - rows

			r at	activity	slack activity	lower limit.	.upper limit.	dual activity	i
	•			3 20000	1 20000			1 0000	
		ob 	ba	3.30000	-3.30000	none	none	1.00000	1
	10		eq	3.00000	.00000	3.00000	3.00000	-1.09999	2
	11		eq	3.82600	.00000	3.82600	3.82600	00001	3
a	12	r 3	eq	3.00000	.00000	3.00000	3.00000	.00000	4
	13	r 4	eq	1.00000	.00000	1.00000	1.00000	0,00000	Б
•	14	r 6	•9	-1.00000	.00000	-1.00000	-1.00000	.00000	6
sect	ion	2 - co]	umns						
nu	nber	.colum	m. at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m+j
	4	x 1	h-1	3 00000	1 10000	~~~~		.00000	7
			bø te	3.00000	1.10000	. 00000	none E 00000		7
		x2	be	1.58457	0.00000	none	5.00000	.00000	8
		x3	11	.00000	.00000	.00000	2.00000	-1.09999	9
		x4	bs	.58700	.00000	.00000	2.00000	00001	10
	5	ж	bs	1.41543	.00000	.00000	2.00000	.00000	11
	6	ක්	ᆈ	2.00000	.00000	.00000	2.00000	0.00000	12
đ	7	x7	bs	2.00000	.00000	.00000	2.00000	.00000	13
	8	rh	eq	-1.00000	.00000	-1.00000	-1.00000	-3.30000	14
•			xr the v	alue of the para	meter= 1	.00000 infinite	range (tmax.ge.1		
data		file							
	12	name		plpr					
	13	'set'		.00	00003+00				
	14			r3 1.0	00004+00	.00000d+0	0		
	15	*							
	16	* K	lote tha	t all components	of rhs vector a	re defined in MP	S file.		
	17	*							
	18	endat	a						
plp	The								
	1100								
					·				
	nber		1		rhs				
	nber 10	r1	1	. 00000		i 2			
	nber	r1	1		3.00000				
	nber 10	r1 r2	.	. 00000	3.00000 3.82600	2			
	10 11 12	r1 r2 r3	1	.00000	3.00000 2 3.82600 2 3.00000 4	2 3			
	nber 10 11	r1 r2 r3 r4	.	.00000 .00000 1.00000	3.00000 2 3.82600 3 3.00000 4 1.00000 4	2 3 4			
	10 11 12 13	r1 r2 r3 r4 r5		.00000 .00000 1.00000 .00000 .00000	3.00000 3.82600 3.00000 1.00000 -1.00000	2 3 4 5 6			
	10 11 12 13	r1 r2 r3 r4 r5		.00000 .00000 1.00000 .00000 .00000	3.00000 2 3.82600 2 3.00000 4 1.00000 4	2 3 4 5 6	variable x5	(5) from 11	
num	10 11 12 13 14	r1 r2 r3 r4 r5	obj=	00000 00000 1.00000 00000 00000 0.36154282d+01	3.00000 3.82600 3.00000 1.00000 -1.00000	2 3 4 5 6 41543d+00	variable x5	(5) from 11	
num pitr rep]	10 11 12 13 14	r1 r2 r3 r4 r5 basic	obj=	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tamax= 0.4	2 3 4 5 6 41543d+00 0 11		(5) from 11	
num pitr rep]	10 11 12 13 14 1= 1 Laces	r1 r2 r3 r4 r5 basic	obj= variabl	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0) which passes to	2 3 4 5 6 41543d+00 o 11 3.6154281824d+		(5) from 11	
num pitr rep] prot stat	10 11 12 13 14 1= 1 Laces	r1 r2 r3 r4 r5 basic	obj≃ variabl test	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0.4) which passes to bjective value	2 3 4 5 6 41543d+00 o 11 3.6154281824d+	00	(5) from 11	
num pitr rep] prot stat	10 11 12 13 14 1= 1 Laces	r1 r2 r3 r4 r6 basic	obj≃ variabl test optimal	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6 .00000	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0.4) which passes to bjective value	2 3 4 5 6 41543d+00 o 11 3.6154281824d+	00	(5) from 11	
num pitr rep] prot stat obje	10 11 12 13 14 14 14 14 14 15 14 14 14 15 14 15 14 15 16 16 17 10 10 11 12 13 14 14 14 14 15 14 14 14 14 14 14 14 14 14 14 14 14 14	r1 r2 r3 r4 r6 basic	obj≃ variabl test optimal ob	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6 .00000	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0.4) which passes to bjective value	2 3 4 5 6 41543d+00 o 11 3.6154281824d+	00	(5) from 11	
num pitr rep] prot stat	10 11 12 13 14 1= 1 Laces blem : ;us ective ;es	r1 r2 r3 r4 r6 basic	obj≃ variabl test optimal ob	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6 .00000	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0.4) which passes to bjective value	2 3 4 5 6 41543d+00 o 11 3.6154281824d+	00	(5) from 11	
num pitr rep] prot stat obje rhs rang bour	10 11 12 13 14 14 14 14 14 14 14 15 14 15 14 15 14 15 16 16 17 17 17 17 17 17 17 17 17 17 17 17 17	r1 r2 r3 r4 r6 basic	obj= variabl test optimal ob rh bo	.00000 .00000 1.00000 .00000 .00000 0.36154282d+01 • x6 (6 .00000	3.00000 : 3.82600 : 3.00000 : 1.00000 : -1.00000 : tmax= 0.4) which passes to bjective value	2 3 4 5 6 41543d+00 o 11 3.6154281824d+	00	(5) from 11	
num pitr rep] prot stat obje rhs rang bour	10 11 12 13 14 14 14 14 14 15 14 14 15 15 16 16 16 16 17 17 17 17 17 17 17 17 17 17 17 17 17	r1 r2 r3 r4 r5 basic name	obj= variabl test optimal ob rh bo	.00000 .00000 1.00000 .00000 0.36154282d+01 • x6 (6 soln i (max)	3.00000 : 3.82600 : 3.00000 / 1.00000 / -1.00000 / tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 6 41543d+00 0 11 3.6154281824d+ superbasics	00 0	(5) from 11	
num pitr rep] prot stat obje rhs rang bour	10 11 12 13 14 14 14 14 14 15 14 14 15 15 16 16 16 16 17 17 17 17 17 17 17 17 17 17 17 17 17	r1 r2 r3 r4 r5 basic name	obj= variabl test optimal ob rh bo	.00000 .00000 1.00000 .00000 0.36154282d+01 • x6 (6 soln i (max)	3.00000 : 3.82600 : 3.00000 / 1.00000 / -1.00000 / tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 6 41543d+00 0 11 3.6154281824d+ superbasics	00 0		
num pitr rep] prot stat obje rhs rang bour	10 11 12 13 14 14 14 14 14 15 14 14 15 16 16 16 16 16 16 16 16 16 16 16 16 16	r1 r2 r3 r4 r5 basic name	obj= variabl test optimal ob rh bo	.00000 .00000 1.00000 .00000 0.36154282d+01 • x6 (6 soln i (max)	3.00000 : 3.82600 : 3.00000 / 1.00000 / -1.00000 / tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 6 41543d+00 0 11 3.6154281824d+ superbasics	00 0		

<pre>n1 r2 eq 3.82800 .0000</pre>									
<pre>13 rd eq 1.00000 10000 1.00000 1.00000 -0.00000 esction 2 - columns mamber .column. stsctivityobj gradientlower limitupper limit. reduced gradnt 1 rd be 2.00000 10000 2.00000 rome .00000 2 rd be 3.11543 1.00000 rome 5.00000 .00000 4 rd 1100000 rome 2.0000000000 6 rd 1100000 rome 2.0000000000 6 rd 1100000 rome 2.0000000000 6 rd 1100000 rome 2.00000 rome 3.0000 6 rd 1100000 rome 2.00000 rome 3.0000 8 rd eq -1.00000 rome 2.00000 rome 3.0000 8 rd eq -1.00000 rome 3.00000 rome 3.00000 8 rd eq -1.00000 rome 3.00000 rome 3.00000 8 rd 1100000 rome 3.0000 rome 3.0000 8 rd eq -1.00000 rome 3.0000 rome 3.0000 8 rd eq -1.00000 rome 3.0000 rome 3.0000 8 rd eq -1.00000 rome 3.0000 rome 3.0000 2 reme 1 rome 3.0000 rome 3.0000 rome 3.0000 rome 3.0000 1 rome 3.0000 rome 3.0000 rome 3.0000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.00000 rome 3.0000 rome 3.0000 rome 3.00000</pre>	11	r2	eq	3.82600	.00000	3.82600	3.82600	77849	3
<pre>a 14 r6 eq -1.0000 .0000 -1.0000 -1.0000 .0000 metian 2 - columns mumber .column atsctuvityobj gradientlower limitupper limit . reduced gradnt i xt be 2.00000 .10000 .00000 .00000 2.00000 .00000 4 xd 1100000 .00000 .00000 2.0000000000 5 xd be 0.00000 .00000 .00000 2.00000</pre>	12	r3	eq	3.41544	.00000	3.41544	3.41544		4
<pre>section 2 - columns mamber .column, stsctivityobj gradientlower limitupper limit. reduced gradint</pre>	13	r 4	eq	1.00000	.00000	1.00000	1.00000	63694	Į
<pre>number .column. atactivityobj gradientlower limitupper limit. reduced gradint</pre>	ı 14	r 6	eq	-1.00000	.00000	-1.00000	-1.00000	.00000	(
<pre>i xi bs 2.0000 .1000 .0000 .xxxs .10000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .20000 .0000 .20000 .7784 .11 .0000 .0000 .0000 .20000 .2 .0000 .0000 .2 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .0000 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .00000 .2 .0000 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .2 .0000 .0000 .0000 .0000 .0000 .2 .0000 .0 .0000 .0000 .0 .0000 .2 .0000 .0 .0000 .2 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0</pre>	ection (2 - column							
2 x2 bs 3.4153 1.0000 nome 5.0000 .0000 3 x3 bs 1.0000 .0000 .0000 2.000077849 5 x5 bs 0.0000 .0000 .0000 2.000077849 5 x5 bs 10 .0000 .0000 .0000 2.00000000 5 x7 x7 bs 1.0000 .00000000 2.00000000 5 x7 x7 bs 1.000000000000 2.00000000 5 x7 s7 bs 1.00000000000000000000 scti - problem is infessible. to and sum of infessible. to and sum of infessible. to and sum of infessible. 1 2 tmore 0.24154401 http://west	number	.column.	at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m +
3 x3 bs 1.00000 .00000 2.00000 .00000 4 x4 11 .00000 .00000 2.00000 .00000 5 x5 bs 0.0000 .00000 2.00000 .00000 6 x5 11 .00000 .00000 2.00000 .00000 8 x5 bs 1.00000 .00000 .00000 .00000 .00000 8 x5 eq -1.00000 .00000 -1.00000 -3.61543 xcit problem is infeasible. set an are of infeasibilities 1 2.573138504-06 . . . xits = 2 tmaye 0.241544-01 adata pipe x2 -1.000004-00 20 'set' .000000 .00000 .00000 .00000 plp bound	1	x1	bs	2.00000	. 10000	.00000	none	.00000	•
<pre>4 xd ll</pre>	2	x2	bs	3.41543	1.00000	ICDO	5.00000	.00000	8
<pre>i 5 x5 be 0.0000 .0000 .0000 2.0000 .0000 6 x6 ll .0000 .0000 .0000 2.0000 .0000 8 rh eq -1.0000 .0000 .0000 2.0000 .0000 8 rh eq -1.0000 .0000 .0000 2.0000 .0000 8 rh eq -1.0000 .0000 .0000 .0000 -1.0000 -3.61543 matt = 2 tmax 0.24154+01 htts plp file 20 'set' .000000+00 22 endsts plp bound plp bound plp bound plp bound i r1 .0000 .0000 .00000 0.0000 2 ii r2 .00000 .00000 .00000 0 section 2 - columns number .column .ll direction .lower limitul direction .upper limit. mrj 1 xd .00000 .00000 .00000 .00000 6 section 2 - columns number .column .ll direction .lower limitul direction .upper limit. mrj 1 xd .00000 .00000 .00000 .00000 8 3 x3 .00000 .00000 .00000 10 5 x5 .00000 .00000 .00000 .00000 .00000 10 5 x5 .00000 .00000 .00000 .00000 .00000 10 5 x5 .00000 .00000 .00000 .00000 .00000 .00000 10 5 x5 .00000 .0000</pre>	3	хЗ	bs	1.00000	.00000	.00000	2.00000	.00000	9
<pre>6 x5 ll .00000 .0000 .0000 2.000063804 7 x7 be 1.0000 .0000 -1.0000 -1.0000 -3.6563 act problem is infeasible. to and sum of infeasible.</pre>	4	x4	11	.00000	.00000	.00000	2.00000	77849	1
7 x7 be 1.0000 .0000 .0000 2.0000 .0000 8 rh eq -1.0000 .0000 -1.0000 -3.61643 ext: problem is infeasible. to and sum of infeasible. pits 2 tmaxe 0.24154+01 Atta plp file 19 name plpb 20 'set'	15	x6	bs	0.00000	. 00000	.00000	2.00000	.00000	1
8 rh eq -1.00000 -1.00000 -1.00000 -3.61643 axit problem is infeasible:	6	хđ	ш	.00000	.00000	.00000	2.00000	63694	1
<pre>mcit problem is infeasible. no. and sum of infeasibilities 1 2.573138800-06 pin= 2 tmax= 0.241540-01 ista plp file </pre>	7	x 7	bs	1.00000	.00000	.00000	2.00000	.00000	1
no. and sum of infeasibilities 1 2.57313880d-06 pits 2 temps 0.24154d+01 hata plp file 	8	rh	۹ą	-1.00000	.00000	-1.00000	-1.00000	-3.61543	1
19 name plpb 20 'set' .0000004+00 21 upper x2 -1.0000004+00 22 endata plp bound member row. .ll direction. .lower limit. .ul direction. .upper limit. .i 10 r1 .00000 .00000 .00000 2 11 r2 .00000 .00000 .00000 2 11 r2 .00000 .00000 .00000 2 12 r3 .00000 .00000 .00000 .00000 4 13 r4 .00000 .00000 .00000 .00000 6 section 2 - columns .unmber .olumn .ll direction .lwer limit. .ul direction .upper limit. m+j 1 xd .00000 .00000 .00000 2.00000 1 2 .x2 .00000 .00000 .00000 2.00000 1 4 <td< th=""><th>uo. and</th><th>sum of in:</th><th>feasib</th><th>ilities 1</th><th>2.57313880d⊣</th><th>06</th><th></th><th></th><th></th></td<>	uo. and	sum of in:	feasib	ilities 1	2.57313880d⊣	06			
<pre>19 name plpb 20 'set'</pre>	iata plp								
21 upper x2 -1.000004+00 .000004+00 22 endsta	19			plpb					
22 endata plp bound section 1 - rows numberrow11 directionlower limitul directionupper limiti 10 ri00000 .00000 .00000 .00000 2 11 r200000 .00000 .00000 .00000 4 13 r400000 .00000 .00000 .00000 4 13 r400000 .00000 .00000 .00000 6 section 2 - columns number .column11 directionlower limitul directionupper limit mej 1 xd00000 .00000 .00000 none 7 2 x200000 none -1.00000 5.00000 8 3 x300000 .00000 .00000 2.00000 10 5 x500000 .00000 .00000 2.00000 10 6 xd00000 .00000 .00000 2.00000 12 7 x700000 .00000 .00000 2.00000 12 6 xd00000 .00000 .00000 2.00000 12 60 change in the optimal solution finite range (tamoa= 0.200004:01)	20	'set'		.00	00+60000				
<pre>plp bound section 1 - rows numberrowll directionlower limitul directionupper limiti 10 ri00000 .00000 .00000 .00000 2 11 r200000 .00000 .00000 .00000 3 12 r300000 .00000 .00000 .00000 4 13 r400000 .000000000000000 6 section 2 - columns number .columnll directionlower limitul directionupper limit m*j 1 xd00000 .00000 .00000 none 7 2 x200000 none -1.00000 5.00000 8 3 x300000 .00000 .00000 2.00000 10 5 x500000 .00000 .00000 2.00000 10 5 x500000 .00000 .00000 2.00000 11 6 x600000 .00000 .00000 2.00000 12 7 x700000 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamon= 0.200004*01) b. no change in the optimal basis</pre>	21	upp	87	x2 -1.0	00001+00	.000000.	0		
<pre>section 1 - rows numberrowll directionlower limitul directionupper limiti 10 rirowll directionlower limitul directionupper limiti 10 rirowll directionlower limitul directionupper limiti 11 r2rowdownorowdownorowdownorowdownorowdownorowdownorowdownorowdownorowupper limitm*j 1 rdrowll directionlower limitul directionupper limitm*j 1 rdrowdownorowdownorowr</pre>	22	endata							
10 r1 .00000 .00000 .00000 2 11 r2 .00000 .00000 .00000 3 12 r3 .00000 .00000 .00000 4 13 r4 .00000 .00000 .00000 5 14 r5 .00000 .00000 .00000 5 section 2 - columns	ection	1 - rows							
11 r2 .00000 .00000 .00000 3 12 r3 .00000 .00000 .00000 4 13 r4 .00000 .00000 .00000 5 14 r5 .00000 .00000 .00000 6 section 2 - columns number .column. .ldirection. .lower limit .uldirection. .upper limit m*j 1 x1 .00000 .00000 .00000 8 3 x3 .00000 .00000 2.00000 9 4 x4 .00000 .00000 2.00000 10 5 x5 .00000 .00000 2.00000 11 6 x4 .00000 .00000 2.00000 12 7 x7 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamoxe= 0.20000d+01)	number	IOW	.11	directionlo	wer limitul	directionup	per limit	.i	
11 r2 .00000 .00000 .00000 3 12 r3 .00000 .00000 .00000 4 13 r4 .00000 .00000 .00000 5 14 r5 .00000 .00000 .00000 6 section 2 - columns number .column. .ldirection. .lower limit .uldirection. .upper limit m*j 1 x1 .00000 .00000 .00000 8 3 x3 .00000 .00000 2.00000 9 4 x4 .00000 .00000 2.00000 10 5 x5 .00000 .00000 2.00000 11 6 x4 .00000 .00000 2.00000 12 7 x7 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamoxe= 0.20000d+01)	10	r1		.00000	.00000	.00000	.00000	2	
13 r4 .00000 .00000 .00000 .00000 6 14 r5 .00000 .00000 .00000 6 section 2 - columns	11	r 2		.00000	.00000	. 00000	.00000	3	
14 r5 .00000 .00000 .00000 6 section 2 - columns number .columnll directionlower limitul directionupper limit m+j 1 xd .00000 .00000 none 2 x2 .00000 .00000 5.00000 3 x3 .00000 .00000 .00000 4 x4 .00000 .00000 .00000 5 x5 .00000 .00000 2.00000 6 x6 .00000 .00000 .00000 7 x7 .00000 .00000 .00000 a. no change in the optimal solution finite range (tamaxa= 0.2000d+01)	12	r3		.00000	.00000	.00000	.00000	4	
section 2 - columns number .columnll directionlower limitul directionupper limit m+j 1 xd .00000 .00000 none 7 2 x2 .00000 none -1.00000 5.00000 8 3 x3 .00000 .00000 .00000 2.00000 9 4 xd .00000 .00000 .00000 2.00000 10 5 x5 .00000 .00000 .00000 2.00000 11 6 x6 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 .00000 13 a. no change in the optimal solution finite range (tamexa= 0.2000d+01) b. no change in the optimal basis	13	r 4		. 00000	.00000	.00000	.00000	5	
number .column, .ll directionlower limit., .ul directionupper limit., m+j 1 x1 .00000 .00000 nome 7 2 x2 .00000 none -1.00000 5.00000 8 3 x3 .00000 .00000 .00000 2.00000 9 4 x4 .00000 .00000 .00000 2.00000 10 5 x5 .00000 .00000 .00000 2.00000 11 6 x8 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 .00000 13 a. no change in the optimal solution finite range (tamoxa= 0.2000d+01) b. no change in the optimal basis	14	тб		. 00000	.00000	. 00000	.00000	6	
1 xd .00000 .00000 nome 7 2 x2 .00000 nome -1.00000 5.00000 8 3 x3 .00000 .00000 .00000 2.00000 9 4 x4 .00000 .00000 .00000 2.00000 10 5 x5 .00000 .00000 .00000 2.00000 11 6 x6 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 .00000 13 a. no change in the optimal solution finite range (tamoxa= 0.2000d+01) b. no change in the optimal basis	section	2 - column	18						
2 x2 .00000 none -1.00000 5.00000 8 3 x3 .00000 .00000 .00000 2.00000 9 4 x4 .00000 .00000 .00000 2.00000 10 5 x5 .00000 .00000 .00000 2.00000 11 6 x6 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 .00000 13 a. no change in the optimal solution finite range (tamoxa= 0.2000d+01) b. no change in the optimal basis	number	.column.	.11	directionlo	wer limitul	directionup	per limit. n	+j	
3 x3 .00000 .00000 .00000 2.00000 9 4 x4 .00000 .00000 .00000 2.00000 10 5 x5 .00000 .00000 .00000 2.00000 11 6 x6 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamoxa= 0.20000d+01)									
4 xd .00000 .00000 2.00000 10 5 x5 .00000 .00000 2.00000 11 6 x5 .00000 .00000 2.00000 12 7 x7 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamexa= 0.20000d+01) b. no change in the optimal basis									
5 x5 .00000 .00000 2.00000 11 6 x8 .00000 .00000 2.00000 12 7 x7 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tamoxa= 0.20000d+01) 13 b. no change in the optimal basis									
6 x6 .00000 .00000 .00000 2.00000 12 7 x7 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tmmxa= 0.20000d+01) b. no change in the optimal basis	-								
7 x7 .00000 .00000 .00000 2.00000 13 a. no change in the optimal solution finite range (tmmxca= 0.20000d+01) b. no change in the optimal basis									
a. no change in the optimal solution finite range (tmaxa= 0.20000d+01) b. no change in the optimal basis									
finite range (tmmon= 0.20000d+01) b. no change in the optimal basis	7	x7		.00000	.00000	. 00000	2.00000	13	
		-	-						
			-						

section 1 - rows

number .	row	.11 d	irection.	.11 boundary a	.11	boundary b	.ul dir	ection.	.ul	boundary a	.ul	boundary b	i
10 r	1		. 00000	.00000		. 00000		.00000		.00000		. 00000	2
11 r	2		.00000	.00000		.00000		.00000		.00000		. 00000	3
12 r	3		. 00000	.00000		.00000		.00000		.00000		.00000	4
13 r	-4		. 00000	.00000		. 00000		.00000		.00000		. 00000	Б
14 r	-6		. 00000	.00000		.00000		.00000		.00000		. 00000	6
section 2	- column	8											
number .	column.	.11 d	irection.	.11 boundary a	. 11	boundary b	.ul dir	ection.	.nl	boundary a	.ul	boundary b	m+j
1 x	d		.00000	.00000		.00000		.00000		none		none	7
2 x	2		. 00000	none		none	-	1.00000		3.00000		3.00000	8
3 ж	ය		.00000	.00000		. 00000		.00000		2.00000		2.00000	9
4 x	c4		. 00000	.00000		.00000		.00000		2.00000		2.00000	10
Бж	æ		. 00000	.00000		.00000		.00000		2.00000		2.00000	11
6 x	ති		.00000	.00000		.00000		.00000		2.00000		2.00000	12
7 x	a		.00000	.00000		. 00000		.00000		2.00000		2.00000	13
pitn= 1	ob	j= 0.	324152784+0	1 tamax=	0.:	20000d+01	Var	iable x5		(5) fra	n 11		
replaces b	masic var	iable	x2 (which pass	es t	o ul							
problem na	me tes	t		objective val	ue	3.241527803	21+00						
status	opt	imal s	nlo	iteration	1	superbasics	0						
			_										
objective	ор		(me:	x)									
rhs	rh												
ranges													
bounds	ьо												
section 1	- IOWS												
n mih an		-+		slack activ	ri+v	lauran lind	.			dual activ			
number .		αι .			109	Iowat 11m		pper rim	10.	. uudi acțiv	109	i	
9 0	ж	bs	3.241	53 -3.24	153	nor		na		1.00		1	
					0000	3.000		3.00			000	2	
a 10 r		eq	3.000							14			
11 r		eq	3.826		0000	3.826		3.82				3	
a 12 r		eq	3.000		0000	3.000		3.00			000	4	
a 13 r		eq	1.000		0000	1.000		1.00			000	5	
а 14 г	5	eq	-1.000	.00	0000	-1.000	000	-1.00	000	.00	000	6	
section 2	- column	8											
_	_												
number .	column.	at .	activity.	obj gradie	nt.	lower limi	itu	pper lim	it.	reduced gra	dnt	ma+j	
			_ ··-									-	
1 3		bs	2.415		0000	.000	000	na		.00		7	
2 x		ul	3.000			nor		3.00		.90		8	
3 3	ය	bs	. 584	57.00	0000	.000	000	2.00	000	.00	000	9	
4 x	c4	п	.000	.00.00	0000	.000	00	2.00	000	14	154	10	
d Ex	æ	bs	0.000	00.00	0000	.000	00	2.00	000	.00	000	11	
6 x	ക	bs	.587	.00	0000	.000	000	2.00	000	.00	000	12	
7 3	a	bs	1.415	43.00	0000	.000	000	2.00	000	.00	000	13	
8 r	th.	eq	-1.000	00.00	0000	-1.000	000	-1.00	000	54	154	14	
		-											
a. no chan	uge in th	e opti	mal solution	n									
	range (t	-	+600000.										
	-												
b. no chan	uge in th	e opti	mal basis										
	range (t	-	0.584574+	00)									
	. .		_										
section 1	- IOMS												
number .		11.8	livection	.11 boundary a	11	boundarar b				houndary a	1	s s	i
				, -			.ui dir	ection.			. u .	. coundary b	· · •

	10 r1		.00000	.00000	.00000	.00000	.00000	.00000	2
	11 r2		.00000	.00000	.00000	.00000	.00000	. 00000	3
	12 r3		.00000	.00000	.00000	.00000	.00000	. 00000	4
	13 r4		.00000	.00000	.00000	.00000	.00000	. 00000	Б
	14 r6		.00000	.00000	.00000	.00000	.00000	. 00000	6
sect	:ion 2 - co	olumns							
num	aber .colu	man11	direction11	boundary a .11	. boundary b .ul	l directionul	boundary a .ul	boundary b	m+j
	1 x1		.00000	.00000	.00000	.00000	none	none	7
	2 x2		.00000	none	none	-1.00000	3.00000	2.41543	8
	3 x3		.00000	.00000	.00000	. 00000	2.00000	2.00000	9
	4 x4		. 00000	.00000	. 00000	. 00000	2.00000	2.00000	10
	5 x5		.00000	.00000	. 00000	.00000	2.00000	2.00000	11
	6 x6		.00000	.00000	. 00000	. 00000	2.00000	2.00000	12
	7 x7		. 00000	.00000	. 00000	.00000	2.00000	2.00000	13
	_								
•	n= 2 Laces basig	-	. 271541314+01		25846d+01	variable x4	(4) from 11	•	
rehi		C VALIADIO) which passes t	.0 UI				
prot	olem name	test	o	bjective value	2.7154131457d	+00			
stat	tus	optimml	soln i	teration 2	superbasics	0			
nhie	ctive	ob	(max)						
rhs		rh	(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,						
rang									
bour	-	Ъо							
sect	ion 1 - ro	5W6							
nun	nberro	ow., at	activity	slack activity	lower limit.	upper limit.	.dual activity	i	
	•	۰.	0.54544	B B (5)(4		
_	9 ob 10 -1	be	2.71541	-2.71541	none	none	1.00000	1	
8	10 r1	eq ••	3.00000	. 00000 . 00000	3.00000	3.00000	.00000	2	
8 8	11 r2 12 r3	eq.	3.82600 3.00000	.00000	3.82600	3.82600 3.00000	.00000.	3	
۵ ۵	13 r4	eq eq	1.00000	.00000	1.00000		.00000	4 Б	
	14 r6	eq eq	-1.00000	.00000	-1.00000		. 10000	6	
		~	1.00000		1.0000	1.00000	. 10000	v	
sect	ion 2 - co	olumns							
מטת	nber .colu	mn. at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m+j	
	1 x1	bs	3.00000	. 10000	.00000	none	.00000	7	
	2 x2	ul	2.41543	1.00000	none	2.41543	1.00000	8	
d	3 x3	be	.00000	.00000	.00000	2.00000	.00000	9	
đ	4 204	bs	0.00000	. 00000	.00000	2.00000	. 00000	10	
	5 x5	bs	. 58457	.00000	.00000	2.00000	.00000	11	
	6 x6	bs	1.41300	.00000	.00000	2.00000	.00000	12	
	7 x7	ul	2.00000	.00000	. 00000	2.00000	. 10000	13	
	8 rh	eq	-1.00000	.00000	-1.00000	-1.00000	10000	14	
	no change i linite rang		immal solution .00000d+00)						
	no change i finite rang		imal basis 0.83086d+00)						
sect	ion 1 - ro	2W6							
משמ	aberre	w11	direction11	boundary a .11	boundary b .ul	l directionul	boundary a .ul	boundary b	i
	10 r1		.00000	.00000	.00000	.00000	.00000	.00000	2

	11 r2		.00000	.00000	. 00000	.00000	.00000	. 00000	3
	12 r3		. 00000	.00000	. 00000	.00000	.00000	. 00000	4
	13 r4		. 00000	.00000	.00000	.00000	.00000	. 00000	5
	14 r5		.00000	.00000	.00000	.00000	.00000	,00000	6
860	tion 2 - c	olumns							
טמ	mber .co]	.uzan11	direction11	boundary a .11	boundary b .1	al direction a	l boundary a .ul	boundary b	m+j
	1 x1		. 00000	.00000	. 00000	.00000	none	none	7
	2 x2		. 00000	none	none	-1.00000	2.41543	1.58457	8
	3 x3		.00000	.00000	. 00000	. 00000	2.00000	2.00000	9
	4 x4		. 00000	.00000	. 00000	.00000	2.00000	2.00000	10
	5 x5		.00000	.00000	. 00000	.00000	2.00000	2.00000	11
	6 x6		.00000	.00000	.00000	.00000	2.00000	2.00000	12
	7 x7		.00000	.00000	.00000	.00000	2.00000	2.00000	13
•	n= 3 Laces basi	•	0.18845506d+01 Le x66 (6	tmax= 0. 3) which passes t	34154d+01 :0 ul	variable x7	(7) from ul		
pro	blem name	test	c	bjective value	1.8845506266	d+00			
sta	tus	optimal	soln	iteration 3	superbasics	0			
obj	ective	ob	(max)						
rhs		rh							
Tan,	•								
bou	nds	ъо							
50 C	tion 1 - 1	CWB							
nu	mber	row at	activity	slack activity	lower limit	upper limit.	.dual activity	i	
	9 ob	bs	1.88455	-1.88455	none	none	1.00000	1	
8	10 r1	eq	3.00000	. 00000	3.0000	0 3.00000	.00000	2	
8	11 r2	eq	3.82600	. 00000	3.8250	0 3.82600	.00000	3	
a	12 r3	eq	3.00000	.00000	3.0000	0 3.00000	.00000	4	
	13 r4	eq	1.00000	.00000	1.0000	0 1.00000	. 14154	Б	
a	14 r5	eq	-1.00000	.00000	-1.0000	0 -1.00000	.00000	6	
800	tion 2 - d	columns							
טת	mber .col	umn. at	activity	.obj gradient.	lower limit	upper limit.	reduced gradnt	m+j	
	1 x1	be	3.00000	. 10000	.0000	0 none	. 00000	7	
	2 x2	ul	1.58457	1.00000	none	1.58457	1.10000	8	
đ	3 x3	be	0.00000	.00000	.0000	0 2.00000	.00000	9	
	4 xx4	be	.58700	.00000	.0000	0 2.00000	. 00000	10	
	5 x65	be	1.41543	. 00000	.0000	0 2.00000	. 00000	11	
	6 x6	미	2.00000	.00000	.0000	0 2.00000	. 14154	12	
đ	7 x7	be	2.00000	.00000	.0000	0 2.00000	.00000	13	
								~	
	8 rh	eq	-1.00000	.00000	-1.0000			14	

minos --- version 4.0 mar 1981 =====

specs file -----

begin Second P L P Test maximize plp order plp rhs analysis 1

plp cost analysis plp bound analysis -3 plp increment 1000. plp frequency 1 data plp file -9 print data plp file and PLP --- version 1.0 june 1986, - - -DEPENDENT WORK of PLP - - ----data plp file ____ 12 name plpr .000000d+00 13 'set' 1.0000001+00 .0000004+00 14 τ3 15 * 16 * Note that all components of rhs vector are defined in MPS file. 17 * 18 endata plp rhs _____ 10 r1 .00000 3.00000 2 .00000 3.82600 11 r2 3 12 r3 1.00000 3.00000 4 1.00000 .00000 13 r4 Б 14 r5 .00000 -1.00000 6 pitn= 1 obj= 0.36154282d+01 tamax= 0.41543d+00 variable x5 (5) from 11 replaces basic variable x6 (6) which passes to 11 problem name test objective value 3.6154281824d+00 iteration 1 superbasics 0 status optimal soln objective (max) οЪ rhs rh TANEOS bounds Ъо section 1 - rows number ...row., at ...activity... slack activity ..lower limit. ..upper limit. .dual activity ..i 9 ob 3.61543 -3.61543 1.00000 be none none 10 **r**1 3.00000 .00000 3,00000 3.00000 .00000 2 eq 11 12 3.82600 .00000 3.82600 3.82600 - .77849 3 eq -.00000 3.42251 3.42251 3.42251 .00000 12 r3 eq -.63694 5 .00000 6 13 r4 1.00000 .00000 1.00000 1.00000 eq 14 гБ eq -1.00000 .00000 -1.00000 -1.00000 section 2 - columns number .column. at ...activity... .obj gradient. ..lower limit. .upper limit. reduced gradnt m+j 2.00000 1 x1 . 10000 .00000 none .00000 7 bs 2 22 bs. 3.41543 1.00000 none 5.00000 .00000

1

4

8

	3	x3	bs	1.00000	.00000	. 00000	2.00000	.00000	9
	4	x4	11	.00000	. 00000	.00000	2.00000	77849	10
d	Б	х£	bs	0.00000	.00000	.00000	2.00000	.00000	11
	6	306	11	.00000	.00000	.00000	2.00000	63694	12
	7	x7	bs	1.00000	.00000	.00000	2.00000	.00000	13
	8	rh	eq	-1.00000	.00000	-1.00000	-1.00000	-3.61543	14

data plp file

••	

1	n.ar	me p	lpc			
2	's(et'		.00000001+00		
3		ĸ	1	1.0000001+00	.00	0004+00
4		ĸ	2	-1.00000d+00	.00	000d+00
Б	*					
6	*	Note:				
7	*	Declaration	of dummy	coefficients (=0 in MPS file)	of the objective
8	*	is not nece	saary bec	use the above o	direction is de	fined in the x1-x2
-						

9 * subspace of cost vectors. 10 * 11 endata

plp cost _____

number.	.colu	un	direction	obj gradient.	m≞+j				
1	x 1		1.00000	.10000	7	,			
	22		-1.00000	1.00000					
3			.00000	.00000					
4			.00000	.00000	-				
-	x6		.00000	.00000					
6	 x66		.00000	.00000					
-	x7		. 00000	.00000					
pitn= 1	L	obi=	0.297848564+01	tmex=	0.4	5000d+00	variable x6	(6) from 11	
•		•	lex3 (
problem	name	test		objective va	lue	2.97848556324	00		
status		optimml	soln	iteration	1	superbasics	0		
objectiv	ne i	ob	(ma)	Ð					
rhs									
T 110		rh							
ranges		rn							
ranges		rh bo							
ranges bounds section		bo	activity	. slack activ	vity	lower limit.	upper limit.	.dual activity	
ranges bounds section number		bo	activity 2.9784		-	lower limit. none	upper <u>lim</u> it. none	.dual activity 1.00000	
ranges bounds section number 9		bo ws w at	2.9784	l9 -2.9	-				
number 9 10	ro ob	bo ws w at bs eq	2.9784	l9 -2.9 10 .0	7849	none	none	1.00000	
number 9 10	ro ob r1	bo ws w at bs eq eq	2.9784 3.0000	19 -2.9 10 .0 10 .0	7849	лоле 3.00000	hane 3.00000	1,00000 00580	
ranges bounds section number 9 10 11 11 12	ro ob r1 r2	bo ws w at bs eq eq eq	2.9784 3.0000 3.8260	19 -2.9 10 .01 10 .01 10 .01	7849 0000 0000	none 3.00000 3.82800	none 3.00000 3.82600 3.42251	1.00000 00580 77849	
number 9 10 11 12 13	ro ob r1 r2 r3	bo ws w at bs eq eq	2.9784 3.0000 3.8280 3.4225	19 -2.9 10 .0 10 .0 11 .0 10 .0	7849 0000 0000 0000	none 3.00000 3.82600 3.42251	none 3.00000 3.82600 3.42251 1.00000	1.00000 00580 77849 .00000	-
number 9 10 11 12 13 14	ro ob r1 r2 r3 r4 r5	bo ws w at bs eq eq eq eq	2.9784 3.0000 3.8260 3.4225 1.0000	19 -2.9 10 .0 10 .0 11 .0 10 .0	7849 0000 0000 0000 0000	none 3.00000 3.82600 3.42251 1.00000	none 3.00000 3.82600 3.42251 1.00000	1.00000 - 00680 - 77849 .00000 .00000	-
ranges bounds section number 9 10 11 12 13 13 14 section	ro ob r1 r2 r3 r4 r5 2 - co	bo wes w at bs eq eq eq eq	2.9784 3.0000 3.8260 3.4225 1.0000 -1.0000	9 -2.9 0 .0 10 .0 11 .0 10 .0 10 .0	7849 0000 0000 0000 0000 0000	none 3.00000 3.82600 3.42251 1.00000 -1.00000	none 3.00000 3.82600 3.42251 1.00000 -1.00000	1.00000 - 00680 - 77849 .00000 .00000	
ranges bounds section number 9 10 11 a 12 a 13 a 14 section number	ro ob r1 r2 r3 r4 r5 2 - co	bo wes w at bs eq eq eq eq	2.9784 3.0000 3.8260 3.4225 1.0000 -1.0000	9 -2.9 10 .0 10 .0 11 .0 10 .0 10 .0 10 .0	7849 0000 0000 0000 0000 0000	none 3.00000 3.82600 3.42251 1.00000 -1.00000	none 3.00000 3.82600 3.42251 1.00000 -1.00000	1.00000 - 00580 - 77849 .00000 .00000 .00000	

3 x3 ц .00000 .00000 .00000 2.00000 -.00580 9 .00000 2.00000 - . 77849 4 xx4 n .00000 .00000 10 5 x55 1.00708 .00000 .00000 2.00000 . 00000 11 bs 6 x6 bs 1.41300 .00000 .00000 2.00000 .00000 12 2.00000 .00000 13 d 7 x7 bs 2.00000 .00000 .00000 8 rh -1.00000 .00000 -1.00000 -1.00000 -2.99590 14 **eq** obj= 0.33000001d+01 (4) from 11 pitn= 2 tmax= 0.10000d+01 variable x4 (6) which passes to ul replaces basic variable xf 3.300000715d+00 problem name test objective value status optimal soln iteration 2 superbasics 0 objective (max) ob rhs rh ranges bounds Ъо section 1 - rows number ...row.. at ...activity... slack activity ..lower limit. ..upper limit. .dual activity ..i 9 ob bs 3.30000 -3.30000 none none 1.00000 1 10 r1 3.00000 .00000 3.00000 3.00000 -1.09717 2 eq. 11 r2 3.82600 .00000 3.82600 3.82600 -.00200 3 eq. 12 r3 3.42251 .00000 3.42251 3.42251 .00000 4 a eq 13 r4 1.00000 .00000 1.00000 1.00000 .00200 5 eq. -1.00000 .00000 -1.00000 -1.00000 00000 14 r5 6 æ eq section 2 - columns number .column. at ...activity... .obj gradient. ..lower limit. ..upper limit. reduced gradnt m+j 1.10000 1 x1 bs 3.00000 .00000 none 0.00000 7 1.58457 0.00000 2 x2 bs 5.00000 . 00000 none 8 3 x3 ц .00000 .00000 .00000 2.00000 -1.09717 9 4 x04 .58700 .00000 .00000 2.00000 -.00200 bs 10 5 x65 1.83793 .00000 .00000 2.00000 .00000 bs 11 6 x6 ul 2.00000 .00000 .00000 2.00000 .00200 12 2.00000 .00000 .00000 2.00000 .00000 đ 7 x7 bs 13 -1.00000 8 rh eq .00000 -1.00000 -1.00000 -3.29717 14

pitn= 3 for the value of the parameter=

1.00000 infinite range (tmax.ge.1.e15)

data plp file

19	name	plpb		
20	'set'		.0000000d+000	
21	upper	x2	-1.00000d+00	.00+b00000.
22	endata			

plp bound

section 1 - rows

number	row	.11 direction.	.lower limit	.ul direction.	.upper limit	1
10	r1	. 00000	.00000	. 00000	.00000	2
11	r2	. 00000	.00000	. 00000	.00000	3
12	r3	.00000	.00000	.00000	.00000	4
13	r 4	.00000	.00000	.00000	.00000	5

14 r 5	. 00000	.00000	. 00000	. 00000	6					
section 2 - columns										
number .co]	umm11 direction.	.lower limit	.ul direction.	upper limit	m +j					
1 x1	. 00000	.00000	.00000	none	7					
2 22	.00000		-1.00000	5.00000	8					
3 x3	.00000		.00000	2.00000	9					
4 x4	.00000		.00000	2.00000	10					
5 x5	.00000		.00000	2,00000	11					
6 36	.00000		.00000	2.00000	12					
7 x7	. 00000		.00000	2.00000	13					
-										
-	in the optimal solut uge (tamecca= 0.34154									
h an channe	in the option? hasis									
-	in the optimal basis uge (tmmodb= 0.34154									
	nke (rumon- 0.94194									
section 1 - 1	TOWE									
number	row11 direction.	.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a	.ul boundary b	i			
10 r1	.00000	.00000	. 00000	.00000	.00000	.00000	2			
11 r2	.00000	.00000	. 00000	. 00000	.00000	.00000	3			
12 r3	. 00000	.00000	. 00000	. 00000	.00000	.00000	4			
13 r4	. 00000	.00000	.00000	. 00000	.00000	. 00000	Б			
14 гБ	.00000	.00000	.00000	. 00000	.00000	.00000	6			
section 2 - (columns									
number .col	lumm11 direction.	.11 boundary a	.11 boundary b	ul direction.	.ul boundary a	.ul boundary b	m+j			
1 11	.00000	.00000	.00000	.00000	noñe	none	7			
2 22	.00000		none	-1.00000	1.58457	1.58457	8			
3 x3	.00000		.00000	.00000	2.00000	2.00000	9			
4 x4	.00000		.00000	.00000	2.00000	2.00000	10			
5 x5	.00000		.00000	.00000	2.00000	2.00000	11			
6 x6	.00000		.00000	.00000	2.00000	2.00000	12			
 7 x7	.00000		.00000	.00000	2.00000	2.00000	13			
pitn= 1	obj= 0.33000001d	+01 tmax=	0.341544+01	variable x3	(3) fra	m 11				
replaces bas:	ic variable x2	(2) which pass	as to ul							
problem name	test	objective valu	ae 3.300000101;	3d+00						
status	optimal soln	iteration :	1 Superbasics	0						
objective	ob (max)								
rhs	rh									
ranges										
bounds	Ъо									
section 1 - rows										
number	row atactivit	y slack activi	itylower limi	tupper lim	itdual activ	ityi				
e e e	bs 3.3	0000 -3.30	000 non	e na	ne 1.00	000 1				
a 10 r1	•q 3.0	.000 .000		00 3.00						
a 11 r2	-	2600 .000								
a 12 r3	-	.000								
13 r4	-	.000								
а 14 гб	•	.000								
	•				,	-				

nu	nber	.column.	at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m+j
	1	x1	bs	3.00000	1.10000	.00000	none	. 00000	7
	2	x2	ul	1.58457	0.00000	none	1.58457	1.10000	8
d	3	хЗ	bs	0.00000	. 00000	.00000	2.00000	. 00000	9
	4	x1	bs	.58700	.00000	.00000	2.00000	.00000	10
	5	хБ	bs	1.83793	.00000	.00000	2.00000	. 00000	11
	6	36	nl	2.00000	. 00000	.00000	2.00000	1.55697	12
d	7	хЛ	bs	2.00000	. 00000	.00000	2.00000	. 00000	13
	8	rh.	eq	-1.00000	. 00000	-1.00000	-1.00000	1.55697	14

a. no change in the optimal solution finite range (tmsom= .00000d+00)

b. no change in the optimal basis finite range (tmandr= 0.16207d+00)

section 1 - rows

number	row	.11 direction.	.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a	.ul boundary b	i
10	ri	.00000	.00000	. 00000	. 00000	.00000	.00000	2
11	r2	. 00000	.00000	.00000	.00000	.00000	.00000	3
12	r3	. 00000	.00000	.00000	.00000	.00000	.00000	4
13	r 4	.00000	.00000	. 00000	.00000	.00000	.00000	Б
14	r6	.00000	.00000	. 00000	.00000	.00000	. 00000	6

section 2 - columns

number	.column.	.11 direction.	.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a	.ul boundary b	∎+j
1	x1	.00000	.00000	.00000	.00000	none	none	7
2	x2	.00000	none	none	-1.00000	1.58457	1.42251	8
3	x3	.00000	.00000	.00000	.00000	2.00000	2.00000	9
4	xx4	.00000	.00000	. 00000	.00000	2.00000	2.00000	10
5	x6	.00000	.00000	.00000	.00000	2.00000	2.00000	11
6	x6	.00000	.00000	.00000	.00000	2.00000	2.00000	12
7	x7	. 00000	.00000	.00000	.00000	2.00000	2.00000	13

wcit -- problem is infeasible.

no. and sum of infeasibilities 1 9.99999046d-03

pitn= 2 tmax= 0.35775d+01