A Standard Input Format for Multiperiod Stochastic Linear Programs

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WORKING PAPER

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FOREWORD

Data conventions for the automatic input of multiperiod stochastic linear programs are described. The input format is based on the MPSX standard and is designed to promote the efficient conversion of originally deterministic problems by introducing stochastic variants in separate files. A flexible "header" syntax generates a useful variety of stochastic dependencies. An extension using the NETGEN format is proposed for stochastic network programs.

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A STANDARD INPUT FORMAT FOR MULTIPERIOD STOCHASTIC LINEAR PROGRAMS[1]

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0. Introduction

The desire to solve stochastic optimization problems as more realistic models than deterministic formulations for decision making began in the the early days of linear programming, see Dantzig [3] for example. As evidenced by recent volumes, Prekopa and Wets [12] and Ermoliev and Wets [5], there is a wide body of active research into numerical algorithms. However, these efforts have tended to be directed towards specialized types of problems resulting in a pot-pourri of numerical codes, problem formulations and data sets that are not comparable or even mutually compatible. The development of general purpose codes suitable for industrial application is hampered by the lack of a comprehensive set of test problems in a standard format to compare performance and challenge competitors. In this paper we present our ideas on what we hope will become the nucleus of a standard problem format for multistage stochastic linear programs.

The present contribution is an extension, simplification, and refinement of the ideas in Edwards, et al [4]. The spirit of the two proposals is identical: (a) to use the MPSX convention for file formats; (b) to allow originally deterministic problems to be changed to stochastic versions; (c) to permit a wide variety of stochastic dependencies; and (d) to allow new options to be added. Two key aspects of the original proposal are retained—the use of separate files to specify parameters related to random variables and the use of "header lines" to express and organize the relevant parameters.

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After a statement of the problem in the form we shall consider it, we present in Section 2 our proposed input format. In the next section of this paper we present an extension of the standard format to cover multistage stochastic network problems using the NETGEN format [9], and in the final section our recommendations for mixed stochastic network and linear programs.

The collection of test problems is an ongoing enterprise in which readers are encouraged to participate. Some examples, augmenting those already presented in King [8] and illustrating the standard problem format, are available in computer accessible form.

1. Problem Statement

A general form of the multiperiod stochastic linear program is:

$$\text{minimize} \quad c_1 x_1 + E_2 Q_2(x_1)$$
$$\text{subject to} \quad x_1 \in \mathbb{R}^n$$
$$\ell_1 \le x_1 \le u_1$$
$$A_1 x_1 = b_1,$$

where the functions $Q_2, Q_3, \ldots, Q_T$ are defined recursively:

$$Q_2(x_1) = \min\{c_2 x_2 + E_3 Q_3(x_1, x_2)$$
$$\text{subject to} \quad x_2 \in \mathbb{R}^{n_2}$$
$$\ell_2 \le x_2 \le u_2$$
$$A_{21} x_1 + A_{22} x_2 = b_2\};$$
$$Q_3(x_1, x_2) = \min\{c_3 x_3 + E_4 Q_4(x_1, x_2, x_3)$$
$$\text{subject to} \quad x_3 \in \mathbb{R}^{n_3}$$
$$\ell_3 \le x_3 \le u_3$$
$$A_{31} x_1 + A_{32} x_2 + A_{33} x_3 = b_3\};$$

and so forth, for $t = 4, \ldots, T - 1$, where "E_t" represents expectation with respect to the random variables in period t, until finally

$$Q_T(x_1, \ldots, x_{T-1}) = \min\{c_T x_T$$
$$\text{subject to} \quad x_T \in \mathbb{R}^{n_T}$$
$$\ell_T \le x_T \le u_T$$
$$A_{T1} x_1 + \cdots + A_{TT} x_T = b_T\}.$$

The data defining this problem may be conveniently arranged in an LP formulation for a single realization of the random variables:

(MP)   objective: $c_1 x_1 + c_2 x_2 + \cdots + c_T x_T$

         constraints: $x_t \in \mathbb{R}^{n_t} \qquad t = 1, \ldots, T$

$$\ell_t \leq x_t \leq u_t \qquad t = 1, \ldots, T$$

$$A_1 x_1 \qquad = b_1 \in \mathbb{R}^{m_1}$$

$$A_{21} x_1 + A_{22} x_2 \qquad = b_2 \in \mathbb{R}^{m_2}$$

$$\cdots$$

$$A_{T1} x_1 + A_{T2} x_2 + \cdots + A_{TT} x_T = b_T \in \mathbb{R}^{m_T}.$$

All entries of the matrices A_{ts} and vectors: c_t, ℓ_t, u_t, b_t may be random (although in practice all but a few entries will be deterministic). The indices $t = 1, \ldots, T$ signify the periods of the problem; to each period t there corresponds a decision vector $x_t \in \mathbb{R}^{n_t}$. The lower block-triangular constraint system expresses the typical feature of these problems—the decisions of the prior periods constrain the decision of the current period explicitly (since those decisions are known) but the decision of the current period is affected only implicitly by the costs of possible future (recourse) decisions.

The proposed format is most easily understood by considering the problem (MP) in stages of gradually increasing levels of detail. First we regard (MP) as a purely deterministic problem and create an input file following the MPSX convention, ignoring (non-random) zero entries. This file will identify an objective vector c, upper and lower bounds u and ℓ, a right hand side b, and a block lower-triangular matrix A, all expressed in the usual column-row format; we call this the *core file*.

Next we note the period structure of the problem, that certain decisions $x_t \in \mathbb{R}^{n_t}$ are made at times $t = 1, \ldots, T$. Here it is necessary only to specify which rows and columns from the core file correspond to which periods. It is most simply done by indicating the beginning column and row for each period t. This is done in the *time file*. Note that such a system relies on the proper sequencing of the core file—we require that the list of row names is in order from first period to last period and that the block lower-triangular matrix A has been entered in column order: first period columns first, and last period columns last.

Finally there remains the specification of the distributions of the random entries; this takes place in the *stoch file*. The simplest case occurs when all random entries are mutually *independent*—one merely indicates by keywords which entries are distributed as uniform, beta, gamma, etc., or discrete and supplies the appropriate parameters. One may even use the entries in the core file to supply some of the parameters. We consider two general types of dependency among the random entries: *blocks* and *scenarios*. A block is a random vector whose realizations are observed in a single, fixed period. A *scenario* is a more general type of stochastic structure which models dependencies across periods. Following Lane and Hutchinson [10], we visualize scenarios as paths in an *event tree*. Each

path corresponds to a particular sequence of events through time, and is assigned a probability that is the product of the separate event probabilities.

The organization of the data files is similar to the MPSX format [6; pp. 199–209]. Each data file contains a number of *sections* some of which are optional. A *header line*, or *header*, marks the beginning of each section and may contain *keywords* to inform the user that the data to follow should be treated in a special way. Each header line is divided into two fields delineated by specific columns.

Header line

- columns 1 through 14: first word field

- columns 15 through 72: second word field

Most sections contain *data lines*, these are differently arranged for linear programs than for networks and each version will be described in the appropriate sections. Data lines are distinguished by a blank in the first column; the first word of a header line must, therefore, begin with a non-blank character. Finally, *comment lines* are indicated by an asterisk (*) in the first column and may appear anywhere.

We close this section with a comment to potential users. The header lines are a powerful device that permits the expression of a wide variety of problem types, however one should not exploit it by tailoring a format convenient for one's own peculiar taste in test problems. That is definitely not in the spirit of our proposal! Rather we hope that this proposal offers sufficient flexibility to be useful and we trust that many problems will become available using the basic elements to the fullest extent—with tailored modifications only when absolutely necessary.

2. Standard Format for Linear Programs

2.1 Core File

The *core file* is sketched only briefly since it closely follows the MPSX standard [6]. The core file consists of sections introduced by header lines. Data lines follow the headers. We assume that all appropriate dimensioning has been done before the files are read (for example in a file similar to the SPECS file for the MINOS program [11]).

A data line is divided into six fields: three name fields, two numeric fields and one code field.

Data line for LP

- columns 2 and 3: code field

- columns 5–12: first name field

- columns 15–22: second name field

- columns 25–36: first numeric field

- columns 40–47: third name field

- columns 50–61: second numeric field

A *name* is treated as a character string and may contain any ASCII symbol. Only numbers with decimal point, or in scientific notation, may appear in numeric fields.

Core file sections for LP

1. NAME – starts the input file. The second word field is used to identify the problem.

2. ROWS section – each data line specifies the names of the objective c and rows of the matrix A in the first name field, and the type of constraint (E, L, G, N) in the code field. The list of row names must be in order from first period to last, preceded by the objective name(s).

3. COLUMNS section – each data line specifies the column names and the nonzero values of c and A. This must be done in column order.

4. RHS section – data lines specify nonzero entries of the righthand side b.

5. BOUNDS (optional) – data lines specify the bound, u and ℓ, using the codes LO or UP in the code field.

6. RANGES (optional) – cf. the MPSX standard [6].

7. ENDATA – informative. End of problem data.

Core file - example

```
NAME              problem name
ROWS
  N     OBJ
  L     ROW1
  E     ROW2
         . . . . . .
COLUMNS
        COL1    ROW1    value    ROW2    value
        COL1    ROW3    value    ROW4    value
                 . . . . . .
        COL2    ROW1    value    ROW2    value
RHS              . . . . . .
        RHS     ROW1    value    ROW2    value
         . . . . . .
BOUNDS
  LO    BND     COL1    value
         . . . . . .
RANGES
         . . . . . .
ENDATA
```

If an entry is *random* then the value field should contain a non-zero number (which may or may not be meaningful). It *must not be omitted* from the core file. Integer variables may be indicated by using delimiters as described in the MPSX/MIP documentation [7] for those users who want to use mixed integer programming techniques.

2.2 Time File

The *time file* contains the information needed to specify the dynamic structure of the problem. It indicates which rows and columns (i.e. elements of the decision vector x) are to be identified with which period. The first line identifies it as a time file and gives a name to the problem. The next header line consists of a single word PERIODS in the first word field, and contains the keyword LP in the second word field to identify the problem as a pure LP. The first two name fields of the data lines identify the beginning row and column names for each period with the corresponding period name in the third name field.

Time file - example

TIME	problem name	
PERIODS	LP	
COL1	ROW 1	PERIOD1
COL6	ROW 3	PERIOD2
COL8	ROW19	PERIOD3
ENDATA		

In this example: Columns COL1 through COL5 are PERIOD1 decision variables and COL6 and COL7 are PERIOD2 variables; rows ROW1 and ROW2 are PERIOD1 constraints and ROW3 through ROW18 are PERIOD2 constraints. All remaining rows and columns are PERIOD3.

2.3 Stoch File

In the *stoch file* the distributions of the random variables are specified. As mentioned, we consider three varieties of distributions: *independent*, *blocks*, and *scenarios*. Each type will be treated in separate sections of this file; each section consists of a header line followed by data lines.

Stoch file - header lines

1. STOCH – informative. Identifies a new problem with a given name in the second word fields.
2. INDEP section – specifies the distribution of all independent random entries in separate sections for each type.
3. BLOCKS section – specifies the joint distribution of all dependent random entries in separate sections for each type.
4. SCENARIOS section – specifies the scenarios.
5. ENDATA – informative. End of problem data.

2.3.1 A note on distributions

The purpose of the stoch file is to give the user the information needed to compute with the random variables. In many applications the distributions are discrete or discrete approximations of continuous distributions; thus the user needs, ultimately, to know what value the random variable takes and with what probability. The discrete case is straightforward—this information may be explicitly provided in the stoch file and then stored in appropriate data structures by the user. In the continuous case users may have their own discretization scheme and may need only to know the parameters and type of the continuously distributed random variables. Such data is easily provided, however users must then process it themselves to obtain a discrete approximation. Finally there are cases where the random variables may be accessed only through a *user-supplied subroutine*—for example the output of a random number generator of nonstandard type. Alternatively, the user may be able to compute directly with certain continuous distributions and may build approximations based on such distributions—for example piecewise linear, or quadratic distributions as investigated by Wets [15] and Birge and Wets [2]. This information is easily transmitted using the various data structures described below.

2.3.2 Independent

Independent random variables are easily treated. We provide facilities for identifying entries that are distributed as discrete, uniform, beta, gamma, normal and log-normal; certainly other distributions may be considered. The INDEP header line is repeated for each new distribution type; entries with the same type are listed together following the appropriate header. The keyword in the second word field of the header identifies the distribution. The data lines indicate the entry by column and row in the first two name fields, and the distribution parameters are entered in the first and second numeric fields.

Discrete. For each discretely distributed entry one must specify the values and corresponding probabilities. The first two name fields identify the entry, and the first two numeric fields are the value field and probability field respectively. The intervening third name field contains the name of the period in which the random variable is realized (this information is redundant but is useful to have made explicit).

INDEP	DISCRETE			
COL1	ROW8	6.0	PERIOD2	0.5
COL1	ROW8	8.0	PERIOD2	0.5
RHS	ROW8	1.0	PERIOD2	0.1
RHS	ROW8	2.0	PERIOD2	0.5
RHS	ROW8	3.0	PERIOD2	0.4

In this example the entry COL1/ROW8 takes value 6.0 with probability 0.5 and 8.0 with probabil-

ity 0.5; and the righthand side of ROW8 takes values $\{1.0, 2.0, 3.0\}$ with probabilities $\{0.1, 0.5, 0.4\}$ respectively. Of course the probabilities associated with an entry must total one.

Uniform. The endpoints of the interval are the only relevant parameters for uniformly distributed entries. These are entered into the first two numeric fields; the third name field is blank.

```
INDEP          UNIFORM
      COL1    ROW8      8.0    PERIOD2    9.0
```

In this example the random entry COL1/ROW8 is uniformly distributed over the interval $[8.0, 9.0]$.

Normal. The normal distribution is specified by mean μ and variance σ^2 in the first two numeric fields.

```
INDEP          NORMAL
      COL1    ROW8      μ     PERIOD2    σ²
```

Beta, Gamma, Lognormal. Since these distributions are two parameter families, they may be handled in a similar fashion to the normal, using the standard descriptors as presented in, for example, Raiffa and Schlaifer [14].

Subroutine. Some random entries may have distributions that are computed by subroutines, for example empirical distributions, which are discretely distributed but whose values may be randomly generated by user-supplied computer codes.

```
INDEP          SUB
      COL1    ROW8    blank    PERIOD2
```

This example indicates that the user must access a subroutine to generate an appropriate distribution for the entry COL1/ROW8.

2.3.3 Blocks

Blocks may be regarded as mutually independent random vectors. We provide for three distribution types: *discrete, subroutine,* or *linear transformation*. As in the independent case, blocks with common distribution types are grouped in the same section under a header line.

Discrete. The "values" of a block are actually vectors of values of the entries that make up the block, and to each value of a block there corresponds a probability. We need two sorts of data lines to describe a block. The first line, distinguished by a "BL" in the code field, gives the name of the block, the name of the period in which the block is realized, and the probability that the block assumes a given vector value; the following lines identify which entries of the block assume which value.

BLOCKS		DISCRETE	
BL	BLOCK1	PERIOD2	0.5
	COL1	ROW6	83.0
	COL2	ROW8	1.2
BL	BLOCK1	PERIOD2	0.2
	COL2	ROW8	1.3
BL	BLOCK1	PERIOD2	0.3
	COL1	ROW6	84.0

One needs to record only those values that change. We adopt the convention that the first statement of the block is the basis from which all changes are computed (thus zero values must be stated explicitly). In this example the block, called BLOCK1, is the 2-vector made up of the entries COL1/ROW6 and COL2/ROW8. It takes values $(83.0, 1.2)$ with probability 0.5, $(83.0, 1.3)$ with probability 0.2, and $(84.0, 1.2)$ with probability 0.3.

Subroutine. The user accesses a subroutine to compute the distribution of the block consisting of the listed entries.

BLOCKS		SUB
BL	BLOCK1	PERIOD2
	COL1	ROW6
	COL2	ROW8
BL	BLOCK2	PERIOD2
	RHS	ROW6
	RHS	ROW8

Here we have identified two blocks, each a 2-vector, BLOCK1 and BLOCK2 whose distributions must be computed by a subroutine.

Linear Transformations. These are blocks whose distribution is computed as a linear transformation of another random vector with independent components, i.e.

$$v = Hu$$

where H is a matrix and v and u are random vectors. The vector v is the block whose distributions is desired; the vector u has independently distributed components of standard type, e.g. normal. We first identify the block as in the subroutine case, then the (marginal) distribution of each (independent) component of u followed immediately by the corresponding column of H.

BLOCKS	LINTR			
BL V	PERIOD2			
COL1	ROW8			
COL3	ROW6			
RV U1	NORMAL	μ	blank	σ^2
HCOL1	HROW1	h_{11}	HROW2	h_{21}
RV U2	UNIFORM	a	blank	b
HCOL2	HROW1	h_{12}	HROW2	h_{22}

This example illustrates the file structure. In this case

$$\text{COL1/ROW8} = h_{11} \cdot N(\mu, \sigma^2) + h_{12} \cdot U(a, b)$$

$$\text{COL3/ROW6} = h_{21} \cdot N(\mu, \sigma^2) + h_{22} \cdot U(a, b).$$

Note that the "names" U1, U2, etc. are irrelevant and may be left blank.

2.3.4 Scenarios

To describe *scenarios* one needs a data structure that expresses inter-period dependencies. This is best developed as a description of the distribution of a *process* vector on the periods, $t = 1, \ldots, T$, just as one may describe a stochastic process in probability theory. We consider the random entries of $(c_t, b_t, A_{ts} : s \leq t)$ as states of a process vector φ_t, for each time $t = 1, \ldots, T$. Given the corresponding (finite dimensional) joint distributions of this process φ, Kolmogorov's construction yields a probability measure P, termed the *process distribution*, on the space Ω of trajectories. Thus, in general, we have the alternatives of describing the distribution of the stochastic process φ in joint or conditional state distribution form, or as a process distribution over trajectories.

More specifically, with the stochastic process φ is associated a *filtration* $\{\mathcal{F}_t : t = 1, \ldots, T\}$, where for each t the sigma algebra $\mathcal{F}_t$ consists of subsets of Ω termed *events* determined by the *history* of the process φ up to time t, and $\mathcal{F}_t \subset \mathcal{F}_{t+1}$ for all $t = 0, \ldots, T - 1$ (with $\mathcal{F}_0 := \{\Omega, \emptyset\}$). Given the process distribution P over the space of trajectories and an event A in period $t + 1$ we may compute the *conditional probability* $P\{\varphi_{t+1} \in A \mid \mathcal{F}_t\}$. Conversely, these conditional probabilities may be composed to generate the finite dimensional distributions of the process φ. In the case under consideration in this paper, all this may be given a much simpler, more graphic, characterization.

To describe the process paths in the discrete state case, note that φ_t can assume only finitely many values for each t. A given history of values φ_s for $s = 1, \ldots, t$ may be followed by a finite collection of values of φ_{t+1}. We think of these as *nodes* in period $t + 1$. Following Lane and Hutchinson [9] and Raiffa [13], we construct an *event tree* representation of the trajectories: Represent the (unique) value of φ_1 by a single node connected by *oriented arcs* from φ_1 to nodes representing the values of φ_2. These are the *descendant* nodes of the first period node in the terminology of Birge [1]. Each node of period 2 is connected to its descendant nodes at period 3 by individual arcs oriented in the

direction of the period 3 nodes. This construction is continued by connecting each *ancestor* at period $t - 1$ with its descendants at period t. Each node of this tree has a single entering arc and multiple departing arcs representing the possible next period events.
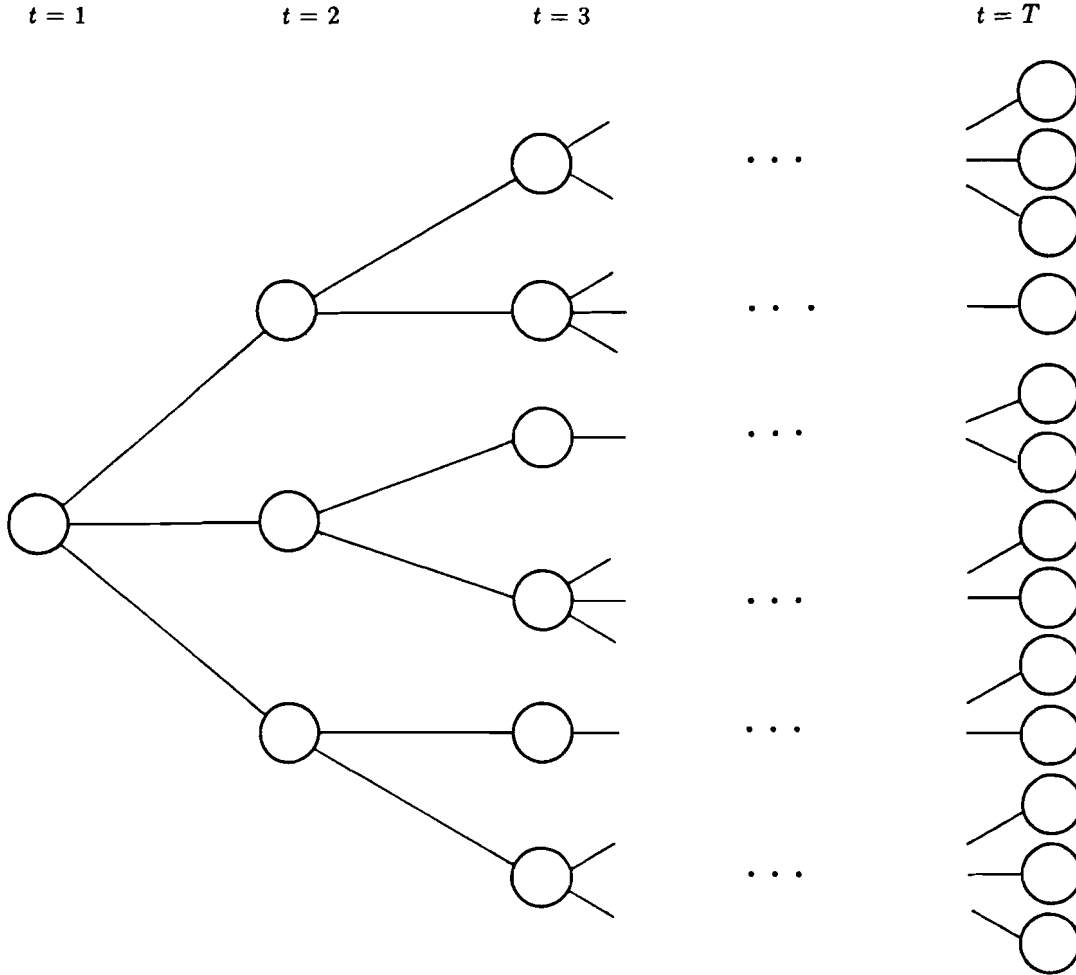

Figure 1. Event tree representation of scenarios

A trajectory φ thus corresponds to a *path* from the period 1 node to a single period T node composed of arcs oriented in the direction of increasing time t and, moreover, corresponds uniquely to a single node in the last period T. There are only finitely many paths linking nodes in period 1 to nodes in period T and to specify the distribution on the paths one needs only to attach a definite probability to each path. Hence specifying the process distribution in terms of path probabilities can be effected in this context by assigning probabilities to period T nodes of the event tree (see Figure 2).

For each period $t = 1, \ldots, T$, the corresponding sigma algebra $\mathcal{F}_t$ is formed by taking all possible unions of the events represented by the nodes of of period t; thus the topology of this event tree represents the information about the process expressed by the filtration $\{\mathcal{F}_t : t = 1, \ldots, T\}$. To each arc we attach the probability that the terminal node occurs given the initial node has occurred—that

is, the conditional probability of φ_{t+1} given the history φ_s for $s = 1, \ldots, t$. These arc probabilities can be computed from the path probabilities by summing the probabilities of the paths visiting the terminal node and then dividing by the sum of the probabilities of the paths visiting the initial node. (The conditional probability $P\{\varphi_{t+1} \in A \mid \mathcal{F}_t\}$ is then represented by the (normalized) distribution over those arcs following nodes in period t whose terminal nodes represent states of φ_{t+1} belonging to A.) Conversely, if to each arc in the tree there corresponds the probability that the terminal node occurs given the initial node has occurred, the probability of any given path is simply the product of the arc probabilities along the path.

(An alternative, perhaps more precise, method of computing the arc probabilities is as follows. To each arc attach a *capacity* of $[0, \infty)$. Place a *source* of 1 at the single first period node and *demands* of value equal to the path probabilities at the corresponding nodes of period T. A *feasible flow* through this capacitated network will assign a unique number to every arc. To obtain the arc probabilities from these flow values one needs only to normalize: for each arc divide the flow value by the total amount of flow arriving at the initial node of the arc. The resulting number is the conditional probability that the terminal node occurs given the initial node has occurred, the arc probability.)

A decision x_t is made only on the basis of information collected up to and including time t. This is represented by a single node in period t. The uncertainty faced by the decision maker is represented by the collection of paths that branch from this node. Thus in the following figure, the decisions occur at the nodes and the scenarios branch after the node.
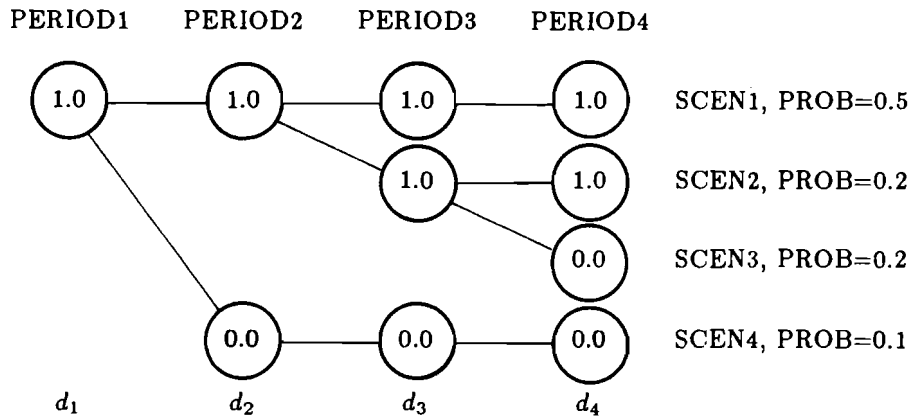
Figure 2.   Scenarios example - event tree

In a language more specific to our application, a "path" in the tree analogy is a single "scenario". The nodes visited by the path correspond to certain values assumed by certain entries of the matrices in the core file. Thus a scenario is completely specified by a list of column/row names and values, and a probability value. Once a single given scenario is described then other scenarios that branch off of it may be described by indicating in which period the branch has occurred, and then listing the subsequent column/row names and values. It is best to work through the example of Figure 2.

There are two types of data lines. The first, signified by SC in the code field, gives the name of

the scenario in the first name field and its probability in the first numeric field; and then gives the name of the scenario from which the branch occurred and the name of the period in which the branch occurred—i.e. the first period in which the two scenarios *differ*—in the second name field and third name field, respectively. A scenario that originates in period one is indicated by ROOT in the name field. The next data lines give the column/row values assumed by the scenario.

Scenarios - example

	SCENARIOS	DISCRETE		
SC	SCEN1	ROOT	0.5	PERIOD1
	COL1	ROW2	1.0	
	COL2	ROW3	1.0	
	COL3	ROW4	1.0	
	COL4	ROW5	1.0	
SC	SCEN2	SCEN1	0.2	PERIOD3
	COL3	ROW4	1.0	
	COL4	ROW5	1.0	
SC	SCEN3	SCEN2	0.2	PERIOD4
	COL4	ROW5	0.0	
SC	SCEN4	SCEN1	0.1	PERIOD2
	COL2	ROW3	0.0	
	COL3	ROW4	0.0	
	COL4	ROW5	0.0	

This is a description of the distribution of 4 entries: COL1/ROW2, COL2/ROW3, COL3/ROW4, COL4/ROW5, which for convenience we denote here as d_1, d_2, d_3, d_4, respectively (see Figure 2). Note that in PERIOD4 there are two nodes for the "state" $d_4 = 0.0$ and two for $d_4 = 1.0$, and similarly in PERIOD3 two nodes for $d_3 = 1.0$. This is because a node is distinguished by the information that one has collected concerning the path up to and including time t. Thus in PERIOD3 the two nodes are distinguished because in scenario SCEN1 one *knows* that the final state is $d_4 = 1.0$, whereas in SCEN2 the outcome of d_4 is in doubt.

3. Network Standard Format

There are several network formats in use. However, it is our view that the most common format is the NETGEN format, see Klingman, Napier and Stutz [9]. For NETGEN we have the following organization of the data line.

Data line for networks

- columns 2–4: code field
- columns 7–12: first name field
- columns 13–18: second name field
- columns 21–30: first numeric field
- columns 31–40: second numeric field
- columns 41–50: third numeric field
- columns 51–60: fourth numeric field

3.1 Core File

As in the MPSX there are header lines and data lines in the input format. We adopt a slight variation of the NETGEN standard here in omitting the BEGIN line and substituting a "NAME" line to start the input file and in changing the END line to ENDATA.

Core file sections for networks

1. NAME - starts the input file. The rest of the line can be used for the problem name.
2. ARCS section - each data line following the ARCS header specifies input for one arc. In the first name field is the name of the *originating node* for the arc, in the second name field the name of the *terminating node*, in the first numeric field the *unit cost*, in the second numeric field the *upper bound* on arc flow, in the third numeric field the *lower bound* on the flow (if not zero) and in the fourth numeric field the *arc multiplier* (arc gain) if we are dealing with generalized networks. If the word UNCAPACITATED follows the ARCS code, upper and lower bounds need not be specified as they are assumed to be ∞ and zero, respectively.
3. SUPPLY (optional) - each data line following the SUPPLY header contains the node name in the first name field and the amount supplied in the second numeric field.
4. DEMAND (optional) - each data line following the DEMAND header contains the node name in the first name field and the amount demanded in the second numeric field.
5. ENDATA - informative. End of problem data.

Note that there is no section naming all nodes (i.e. rows). They are named implicitly by their appearance in the ARCS section. Also note that arcs (i.e. columns) have no names. Hence they cannot be referred to by name, only by a pair of node names. However, if there are parallel arcs, the user must be careful.

Core file - example

```
NAME    problem name
ARCS
        NODE1   NODE2   cost    upper   lower   multiplier
```

NODE1	NODE3	cost	upper	lower	multiplier
. . . . . .					
NODEk	NODEn	cost	upper	lower	multiplier

SUPPLY

. . . . . .			
NODE1		amount	

DEMAND

. . . . . .

ENDATA

3.2   Time File

It is normally assumed that in a NETGEN file, all arcs originating in a given node are given before we start giving arcs originating in the next node. This rule should be followed. Furthermore we shall assume that FROM-nodes are given node order, in the sense that if the arcs originating in NODEk occur before those originating in NODEl, then NODEl belongs to the same or a later time period. However, note that we are not able to give arcs in arc order at the same time. The keyword NETWORK in the PERIODS header indicates that the problem is a pure network.

Time file - example

TIME	problem name
PERIODS	NETWORK
NODE1	PERIOD1
NODE3	PERIOD2
NODE7	PERIOD3

ENDATA

In the example, NODE1 and NODE2 are first period nodes, NODE3 to NODE6 are second period nodes, and all remaining nodes are third period nodes. Arcs going from first to second period nodes are a part of the first period decisions, and carry flow over to the second period by defining external flows for that period.

3.3   Stoch File

Since arcs are defined by *pairs* of nodes, to say that the cost of the arc from node NODE1 to node NODE2 is random requires three parameters. The same goes for bounds and multipliers. We can use the CODE field for that purpose as in the following example. (The data line format used in the stoch file for the networks case is the same as that for the LP case.)

INDEP		DISCRETE	NETWORK			
C1	NODE1	NODE3	6.0		PERIOD2	0.6
C1	NODE1	NODE3	8.0		PERIOD2	0.4
U2	NODE6	NODE8	7.0		PERIOD2	0.2
U2	NODE6	NODE8	9.0		PERIOD2	0.8
SU	NODE6		6.0		PERIOD2	0.1
SU	NODE6		8.0		PERIOD2	0.9

The keyword NETWORK tells you to look for something in the code field. We use C1 for "cost of first arc from node NODE1 to node NODE2" and use higher numbers, such as C2, for the second parallel arc, etc. Similarly, M = multiplier, U = upper bound, and L = lower bound. Random supply and demand are accomodated by use of SU and DE, respectively, in the code fields as illustrated. This format can be changed to all the other distribution forms in obvious ways.

4. Coupled LP and Network Formats

A network can be viewed as a special case of a linear program, yet it is desirable to use the more compact network data format to express those parts of a multistage problem that may be interpreted as network flow problems. The nodes of a network flow problem are the rows in its LP statment. Our proposal for the coupled format utilizes an MPSX-like format, but uses the ARCS card to indicate that the following data lines are in the NETGEN format and the COLUMNS card to indicate that the following lines are in MPSX format. While this proposal is consistent with the philosophy as expressed in the introduction, it does have a particular disadvantage. Many multistage mixed LP/network problems will actually be composed from separate problem files. To generate the proposed format will involve a certain amount of editing of these files, however this editing could be automated in various (system specific) ways.

The first section is the ROWS. (The absence of a ROWS card tells the program that the problem to follow is a pure network.) Note that only those node names that are going to appear as row names of an LP variable need to be named in the ROWS section. Then follows the COLUMNS/ARCS section. Once the data for the columns and arcs has been entered, then the other sections (RHS, BOUNDS, SUPPLY, etc.) may follow. With the proper logic, the same computer program can read all three formats and does not need to be told beforehand the nature of the problem in the file, whether pure LP, pure network, or mixed. As always, the end of the problem is indicated by an ENDATA line. Below is an example for the case when we first have LP, then networks and then LP again.

Core file - example of coupled format

```
NAME                 problem test
ROWS
        . . . . . .
    E       ROW3
    E       ROW4
    L       NODE1
    G       NODE2
    E       ROW5
    E       ROW6
        . . . . . .

COLUMNS
        . . . . . .
        COL16    ROW3     value    NODE1    value
        COL17    ROW4     value    NODE2    value
ARCS
        . . . . . .
        NODE1    NODE2    cost     upper    lower    multiplier
        NODE1    NODE7    cost     upper    lower    multiplier
        NODE2    NODE4    cost     upper    lower    multiplier
        NODE2    NODE7    cost     upper    lower    multiplier
        NODE3    NODE5    cost     upper    lower    multiplier
        . . . . . .
        NODE8    ROW5     cost     upper    lower    multiplier
        NODE8    ROW6     cost     upper    lower    multiplier
COLUMNS
        COL18    ROW5     value    ROW7     value
        COL19    ROW6     value    ROW8     value
        . . . . . .
RHS
        . . . . . .
SUPPLY
        . . . . . .
ENDATA
```

If we now have a TIME file as follows, the coupling is done.

Time file

```
TIME             problem name

PERIODS          MIXED
        COL1     ROW1            PERIOD1

        COL6     ROW3            PERIOD2

                 NODE1           PERIOD3

                 NODE3           PERIOD4

        COL18    ROW5            PERIOD5
ENDATA
```

That the problem is a mixed LP/network is indicated by the keyword MIXED. Here we have that COL1 through COL5 are first stage decision variables, with ROW1 and ROW2 first stage constraints.

COL6 through COL17 are second stage decision variables, with ROW3 and ROW4 as constraints. Arcs originating in NODE1 and NODE2 are third stage decision variables, arcs originating in nodes NODE3 through NODE8 belong to stage 4. Finally, all variables corresponding to columns COL18 through whatever the second COLUMNS section dictates are fifth stage decisions with all constraints after ROW5 associated with them.

The last two entries in the first COLUMNS section will take values from the second to the third stage. The amount will be determined by the values of COL16 and COL17 and the corresponding entries in this COLUMNS section.

The last two entries in the ARCS section will bring "flow" from NODE8 to the right hand side of ROW5 and ROW6 by entering a number in those rows. The number will be the "multiplier" from the input, and the value ending up on the right hand side will be the product of this multiplier and the flow running out of NODE8 to these rows (which are nodes when viewed from the network).

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