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**REPEATED RESUSCITATION:  
HOW LIFESAVING ALTERS LIFETABLES**

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## **Abstract**

How does saving lives affect the force of mortality and life table statistics? How can the progress being made in reducing the force of mortality be interpreted in terms of lifesaving? How many times can a person expect to have his or her life saved as a result of this progress? We develop a model to answer these questions and illustrate the results using U.S. mortality rates for 1900 and 1980 and as projected for 2050.

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## REPEATED RESUSCITATION: HOW LIFESAVING ALTERS LIFETABLES

*James W. Vaupel and Anatoli I. Yashin*

### Introduction

Progress in reducing mortality can be conceived in two ways. Demographers generally view mortality change as change in the force of mortality and associated lifetable statistics for a population.<sup>1</sup> Most laypersons, on the other hand, especially physicians and other health and safety personnel, perceive a reduction in mortality as being achieved by saving the lives of individuals faced with death. A demographer might report that the force of mortality at age fifty among U.S. males was cut in half from 1900 to 1980, from 1.6 percent to 0.8 percent. A public health specialist might focus attention on the lives that were saved in 1980 compared with 1900 because of new surgical and medical procedures, the introduction of penicillin, polio vaccines, and other pharmaceuticals, better nutrition and sanitation, improved automotive safety, less cigarette smoking, faster and more effective ambulance service, and so on.

These two conceptions are not contradictory: both have validity; both aid understanding. Furthermore, both models are abstractions. Demographers are so accustomed to thinking about the force of mortality that they sometime forget how far removed this construct is from the empirical counts of deaths and population numbers on which it is ultimately based. The notion of lifesaving is also an idealization. A lifeguard may believe he saved a swimmer from drowning and a surgeon may believe an operation averted death, but even in these cases there is uncertainty about what would have happened otherwise, and this uncertainty increases when the lifesaving is attributed to, say, better nutrition or healthier lifestyles.

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<sup>1</sup>We assume the reader is familiar with demographic terminology and definitions; for an introductory account, see Keyfitz (1985).

If, however, progress is achieved against mortality, then it seems reasonable to say that lives have been saved, i.e., that deaths have been averted, even though the identity of the individuals saved and the cause of the lifesaving may not be known or even knowable.

In this paper we develop a model that combines the analytical power of the concept of the force of mortality with the appeal and relevance of the notion of lifesaving. We explore both how lifesaving alters life table statistics and, conversely, how change in a life table, in particular change resulting from a reduction in the force of mortality at all ages, can be interpreted in terms of lifesaving. If lives are saved, how will the force of mortality change? If the force of mortality is reduced, how many lives will be saved?

### A Demographic Model of Lifesaving

Let  $\mu(x)$  represent the force of mortality at age  $x$ . Let  $l(x)$  represent survivorship (with a radix of one):

$$l(x) = e^{-\int_0^x \mu(t) dt} \quad (1)$$

Let  $f(x)$  denote the density distribution of deaths:

$$f(x) = \mu(x)l(x) \quad (2)$$

And let  $e(x)$  represent life expectancy:

$$e(x) = \frac{\int_x^\omega l(t) dt}{l(x)} \quad (3)$$

where  $\omega$  is an age beyond which no one survives.

Suppose that the force of mortality is reduced to a new level  $\mu'(x)$  such that

$$\mu'(x) \leq \mu(x) \quad , \quad \text{all } x \quad (4)$$

Let  $l'(x)$ ,  $f'(x)$ , and  $e'(x)$  represent survivorship, the density distribution of deaths, and life expectancy under the new mortality regime. The reduction in the force of mortality can be defined by either

$$\mu'(x) = \mu(x) - \lambda(x) \quad , \quad \lambda(x) \geq 0 \quad \text{all } x \quad , \quad (5)$$

or

$$\mu'(x) = (1 - \delta(x))\mu(x) \quad , \quad \delta(x) \geq 0 \quad \text{all } x \quad , \quad (6)$$

where  $\lambda$  measures the absolute reduction in the force of mortality and  $\delta$  measures the relative reduction,

$$\delta(x) = \frac{\Delta\mu(x)}{\mu(x)} = \frac{\mu'(x) - \mu(x)}{\mu(x)} \quad . \quad (7)$$

Note that the two variables are related by

$$\lambda(x) = \delta(x)\mu(x) \quad . \quad (8)$$

Consider the model depicted in Figure 1. Everyone initially starts off in the leftmost box. From each box the force of mortality is  $\mu'(x)$ , so the overall force of mortality must also be  $\mu'(x)$ . There is a  $\lambda(x)$  intensity of transition to the next box. Because  $\mu(x)$  equals  $\mu'(x)$  plus  $\lambda(x)$ , the rate of attrition from each box is simply the old force of mortality  $\mu(x)$ . Thus, the transition from one box to the next can be considered to represent lifesaving. Under the old mortality regime, the force of mortality was  $\mu(x)$ : under the new regime, this  $\mu(x)$  is divided into two parts, a new force of mortality  $\mu'(x)$  and a force of lifesaving  $\lambda(x)$ . At each age,  $\delta(x)$  of the individuals who would have died are now resuscitated and given another chance.

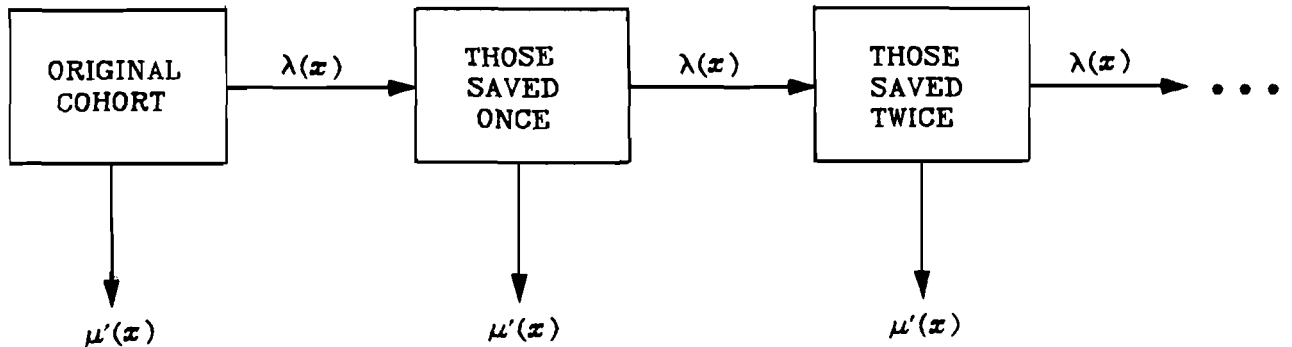


Figure 1. A model of lifesaving.

Individuals can be saved any number of times: the various boxes from left to right include individuals resuscitated zero, one, two, three, and so on, times. The model assumes that a resuscitated individual faces the same life chances as an individual who has not been saved. This assumption simplifies the analysis and thus aids understanding and insight.



Let  $l_i(x)$  denote the probability that an individual will be alive and in state  $i$  at age  $x$ , where  $i$  represents the number of times the individual's life has been saved,  $i = 0, 1, 2, \dots$ . For  $i$  greater than zero, the  $l_i$ 's pertain to revival-survivorship or "revivorship". Because the total force of attrition from state zero is simply  $\mu(x)$ , the value of  $l_0(x)$  equals  $l(x)$ : i.e., survivorship in the first box is the same as survivorship under the old mortality regime. Thus,

$$l'(x) = l(x) + l_1(x) + l_2(x) + \dots \quad (9)$$

If formulas could be found for the revivorship statistics  $l_i(x)$ , then  $l'(x)$ , survivorship under the new mortality regime, could be related to  $l(x)$ , survivorship under the old regime. Furthermore, formulas (2) and (3) could then be used to analyze the change in the distribution of deaths and in life expectancy.

### The Chances of Repeated Resuscitation

Our central result is thus the derivation of formulas for the  $l_i(x)$ , i.e., of the probability that an individual will be resuscitated  $i$  times before ultimate death. The formulas turn out to be remarkably simple:

$$l_i(x) = \frac{l(x)\Lambda(x)^i}{i!}, \quad i = 0, 1, 2, \dots \quad (10)$$

where

$$\Lambda(x) = \int_0^x \lambda(t) dt \quad (11)$$

It follows from (1) and (5) that

$$\Lambda(x) = \ln\left(\frac{l'(x)}{l(x)}\right) \quad (12)$$

In the special case where progress against mortality is uniform at all ages,

$$\delta(x) = \delta, \quad \text{all } x, \quad (13)$$

it is not difficult to show that formula (12) reduces to

$$\Lambda(x) = -\delta(x) \ln l(x) \quad (14)$$

The quantity  $\Lambda(x)$ , which plays such a fundamental role in our results, can be interpreted as the cumulative hazard averted or the cumulative intensity of lifesaving.

Formula (10) can be proven as follows. The differential equation describing  $l_0(x)$  is:

$$\frac{dl_0(x)}{dx} = -\mu(x)l_0(x) \quad . \quad (15)$$

The differential equation has the familiar solution

$$l_0(x) = l_0(0)e^{-\int_0^x \mu(t)dt} \quad . \quad (16)$$

When the radix  $l_0(0)$  is one, as assumed here, this reduces to the desired result:

$$l_0(x) = l(x) \quad . \quad (17)$$

The differential equation describing  $l_1(x)$  is

$$\frac{dl_1(x)}{dx} = -\mu(x)l_1(x) + \lambda(x)l_0(x) \quad . \quad (18)$$

Given the solution for  $l_0$  it is not difficult to show that:

$$l_1(x) = c + l_0(0)\Lambda(x)l_0(x) \quad , \quad (19)$$

where  $c$  is a constant that has to be zero to satisfy the equation at age zero. Setting  $l_0(0)$  equal to one and substituting (17) yields the desired result.

Proceeding iteratively in the same manner, it is possible to solve the differential equation for  $l_2(x)$ ,  $l_3(x)$ , and so on, in turn. These differential equations have the form

$$\frac{dl_i(x)}{dx} = -\mu(x)l_i(x) + \lambda(x)l_{i-1}(x) \quad , \quad i = 2, 3, \dots \quad (20)$$

and their solutions have the form

$$l_i(x) = l(x) \int_0^x \lambda(t_1) \int_0^{t_1} \lambda(t_2) \cdots \int_0^{t_{i-1}} \lambda(t_i) dt_i \cdots dt_2 dt_1 \quad . \quad (21)$$

The nested integrals are readily reduced to  $\frac{\Lambda(x)^i}{i!}$ , completing the proof.

### Revivorship from 1900 to 2050

It follows from (10) that the relationship between survivorship under the new and old mortality regimes, as given by (9), can be rewritten as:

$$l'(x) = l(x) + l(x)\Lambda(x) + \frac{l(x)\Lambda(x)^2}{2} + \dots + \frac{l(x)\Lambda(x)^i}{i!} + \dots \quad (22)$$

or

$$\frac{\Delta l(x)}{l(x)} = \frac{l'(x) - l(x)}{l(x)} = \sum_{i=0}^{\infty} \frac{\Lambda(x)^i}{i!} \quad (23)$$

By analogy with the Poisson distribution, the sum in (23) can be determined and change in survivorship can be summarized as:

$$\frac{\Delta l(x)}{l(x)} = e^{\Lambda(x)} - 1 \quad (24)$$

This expression can also be derived directly from (12). For our present purposes, however, it is the decomposition of lifesaving in (22) and (23) that is of interest.

To illustrate this decomposition, survivorship statistics for U.S. females are presented in Table 1 for three pairs of mortality regimes: 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900. The values of  $l'(x)$  and  $l(x)$  were taken from Faber (1982); the values of  $\Lambda(x)$  were computed, using (12), from these  $l'(x)$  and  $l(x)$  values. The values of the various revivorship statistics  $l_i(x)$  were then calculated using (22). It can be seen from the table that at 1980 mortality rates, some 18933 females out of a birth cohort of 100,000 would survive to age 90, compared with only 1719 individuals at 1900 rates. Hence, some 17214 lives were saved. Of these resuscitated women, 4124 had their life saved once, 4947 had their life saved twice ... and 1813 had their life saved at least five times.

Note that although the progress achieved in saving lives is additive, the breakdown of this progress by number of resuscitations is not additive. The progress achieved from 1900 to 1980 saved the lives of 1478 women who went on to become centenarians and progress from 1980 to 2050 is forecast to save the lives of 10722 women who eventually reach age 100; altogether, 12200 more women (1478 plus 10722) will become centenarians at 2050 rates than at 1900 rates. However, at 1980 rates relative to 1900 rates, only 609 centenarians had their lives five times or more and at 2050 rates relative to 1980 rates, only 346 centenarians benefited so much from lifesaving. But at 2050 rates relative 1900 rates, fully 6585 centenarians, about two-thirds of all the centenarians, will have been

Table 1. Breakdown of the difference in survivorship at selected ages for U.S. females at 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900 mortality rates, with a radix of 100,000.

Comparison	Age	$l'$	$l$	$l' - l$	$l_1$	$l_2$	$l_3$	$l_4$	$l_{5+}$
1980 vs. 1900	10	98521	79646	1875	16939	1801	128	7	0
	30	97495	70969	26526	22537	3578	379	30	2
	50	93872	57711	36161	28075	6829	1107	135	14
	70	75637	31986	43651	27529	11846	3398	731	146
	90	18933	1719	17214	4124	4947	3956	2373	1813
	100	1501	23	1478	96	201	280	292	609
2050 vs. 1980	10	99199	98521	678	676	2	0	0	0
	30	98423	97495	928	924	4	0	0	0
	50	96296	93872	2424	2393	31	0	0	0
	70	83270	75637	7633	7272	350	11	0	0
	90	39994	18933	21061	14159	5294	1320	247	42
	100	9221	1501	7720	2725	2473	1497	679	346
2050 vs. 1900	10	99199	79646	19553	17485	1919	140	8	0
	30	98423	70969	27454	23209	3795	414	34	2
	50	96296	57711	38585	29547	7564	1291	165	18
	70	83270	31986	51284	30604	14641	4669	1117	253
	90	39994	1719	38275	5410	8512	8929	7025	8399
	100	9221	23	9198	138	413	825	1237	6585

reprieved from death at least five times. Some of these women would have been saved three times because of the progress achieved from 1900 to 1980 and an additional two times because of the progress from 1980 to 2050: it is the existence of such multiple lifesaving paths between 1900 to 2050 that explains why so many women will be saved so many times as mortality is reduced from 1900 levels to 2050 levels.

### Postponing Deaths

The density distribution of deaths (or, alternatively, of lifespans), which we call  $f(x)$ , is given by the product of  $\mu(x)$  and  $l(x)$ . It follows from (22) that

$$f'(x) = (1 - \delta(x))f(x)[1 + \Lambda(x) + \frac{\Lambda(x)^2}{2} + \cdots + \frac{\Lambda(x)^i}{i!} + \cdots] \quad (25)$$

This formula could be used to break deaths down into first deaths, second deaths,

and so on, in the sense that second deaths occur among those reprieved once.

The proportion of deaths that occur from each state depends simply on the proportion of surviving individuals in each state. Formulas (10) and (12) imply that the proportion of surviving individuals in each state is given by

$$\pi_i(x) = \frac{l_i(x)}{l'(x)} = e^{-\Lambda(x)} \frac{\Lambda(x)^i}{i!} . \quad (26)$$

The proportions presented in Table 2 were calculated using this formula, with the values of  $\Lambda(x)$  computed, using (12), from available survivorship statistics. As might be expected, hardly anyone is saved from death more than once before age 10, but the proportion benefiting from repeated resuscitation grows with age. In particular, some 71.4 percent of the centenarians alive in 2050 (and 71.4 percent of the centenarians who die in 2050) will have been reprieved five or more times from the deaths they would have suffered at 1900 rates.

Table 2. Breakdown of the proportions of those alive at selected ages who have been resuscitated 0, 1, 2, 3, 4, or 5 or more times, for U.S. females at 1980 vs 1900, 2050 vs. 1980, and 2050 vs. 1900 mortality rates.

Comparison	Age	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_{5+}$
1980 vs. 1900	10	0.808	0.172	0.018	0.001	0.000	0.000
	30	0.728	0.231	0.037	0.004	0.000	0.000
	50	0.615	0.299	0.073	0.012	0.001	0.000
	70	0.423	0.364	0.157	0.045	0.010	0.002
	90	0.091	0.218	0.261	0.209	0.125	0.096
	100	0.015	0.064	0.134	0.186	0.195	0.406
2050 vs. 1980	10	0.993	0.007	0.000	0.000	0.000	0.000
	30	0.991	0.009	0.000	0.000	0.000	0.000
	50	0.975	0.025	0.000	0.000	0.000	0.000
	70	0.908	0.087	0.004	0.000	0.000	0.000
	90	0.473	0.354	0.132	0.033	0.006	0.001
	100	0.163	0.296	0.268	0.162	0.074	0.038
2050 vs. 1900	10	0.803	0.176	0.019	0.001	0.000	0.000
	30	0.721	0.236	0.039	0.004	0.000	0.000
	50	0.599	0.307	0.079	0.013	0.002	0.000
	70	0.384	0.368	0.176	0.056	0.013	0.003
	90	0.043	0.135	0.213	0.223	0.176	0.210
	100	0.002	0.015	0.045	0.090	0.134	0.714

It follows from (25) and (24) that

$$\frac{\Delta f(x)}{f(x)} = \frac{f'(x) - f(x)}{f(x)} = (1 - \delta(x))e^{\Lambda(x)} - 1 \quad (27)$$

and then from (12) that

$$\frac{\Delta f(x)}{f(x)} = (1 - \delta(x)) \frac{\Delta l(x)}{l(x)} - \delta(x) \quad (28)$$

Because death is never avoided but merely postponed, a life saved today will contribute an extra death in the future. Hence the value of  $f'(x)$  has to eventually exceed the value of  $f(x)$ . Both (27) and (28) are consistent with this, although the necessity of a crossover is perhaps most evident from (27). Initially  $\Lambda(0)$  is zero, so the expression in (27) must be negative. If there is no age  $x_0$  after which the value of  $\delta(x)$  never exceeds zero, then  $\Lambda(x)$  increases indefinitely: consequently, the value of the expression must in time become positive. On the other hand, if there is an age  $x_0$  after which  $\delta(x)$  stays at zero, then the expression must be positive after this age.

To illustrate this crossover it is convenient to consider the simple case where  $\delta(x)$  equals  $\delta$  for all  $x$ , i.e., a constant proportion of deaths are averted at all ages. In this case, it can readily be shown that

$$\frac{\Delta l(x)}{l(x)} = l(x)^{-\delta} - 1 \quad (29)$$

and hence, using (28), that

$$\frac{\Delta f(x)}{f(x)} = (1 - \delta)l(x)^{-\delta} - 1 \quad (30)$$

A crossover such that  $f'(x)$  begins to exceed  $f(x)$  occurs at the age  $x^0$  where the expression in (30) equals zero. This age is the age such that

$$l(x^0) = (1 - \delta)^{1/\delta} \quad (31)$$

Thus, if  $\delta$  is 0.5, then  $l(x^0)$  equals 0.25. For U.S. males at 1980 rates, survivorship is down to 25 percent at age 82: if male death rates were cut in half, deaths under the new regime would start exceeding deaths at 1980 rates after age 82.

### Decomposing Life Expectancy

Let  $\tau_i$  denote life years lived in resuscitation state  $i$ :

$$\tau_i = \int_0^{\omega} l_i(x) dx = \frac{\int_0^{\omega} l(x) \Lambda(x)^i dx}{i!} . \quad (32)$$

Then the new value of life expectancy at birth can be decomposed as

$$e'(0) = e(0) + \tau_1 + \tau_2 + \dots \quad (33)$$

and the relative change in life expectancy can be represented as

$$\frac{\Delta e(0)}{e(0)} = \frac{\sum_{i=0}^{\infty} \tau_i}{\int_0^{\omega} l(x) dx} . \quad (34)$$

Table 3 presents a breakdown of the change in life expectancy from one mortality regime to another, for nine different pairs of regimes. Life expectancies in 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900 are compared for males as well as for females. In addition, comparisons are drawn between male and female life expectancies at the mortality rates in 1900, 1980, and 2050: the male/female analysis is possible because male mortality rates either equal (for all practical purposes) or exceed female rates at all ages in each of the three years. In the comparison of 2050 with 1900, note that a significant amount of life expectancy, nearly a quarter of a year for females, is added by saving lives nine times: a cat may have nine lives, but progress in reducing mortality will give some humans at least ten. In the comparison of female and male life expectancies, most of the additional female life expectancy is attributable to saving lives once: if males could gain that extra life females enjoy, the gap between male and female life expectancy would be cut by 95 percent, 80 percent, and 79 percent at 1900, 1980, and 2050 rates, respectively.

In the simple case where the same proportion of deaths are averted at all ages, (34) can be reexpressed as

$$\frac{\Delta e(0)}{e(0)} = \delta H_1 + \delta^2 H_2 + \delta^3 H_3 + \dots , \quad (35)$$

where

Table 3. Breakdown of life expectancy  $e'(0)$  into its components  $e(0)$ ,  $\tau_1, \tau_2, \dots, \tau_9$ , for U.S. females and males at 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900 mortality rates and for females vs. males at 1900, 1980, and 2050 rates.

Comparison	$e'(0)$	$e(0)$	$\tau_1$	$\tau_2$	$\tau_3$	$\tau_4$	$\tau_5$	$\tau_6$	$\tau_7$	$\tau_8$	$\tau_9$
Females, 1980 vs. 1900	77.53	49.07	19.84	5.89	1.79	0.60	0.21	0.08	0.03	0.01	0.01
Males, 1980 vs. 1900	69.85	46.56	17.91	4.28	0.87	0.18	0.04	0.01	0.00	0.00	0.00
Females, 2050 vs. 1980	83.84	77.53	4.83	1.01	0.31	0.11	0.04	0.01	0.00	0.00	0.00
Males, 2050 vs. 1980	75.84	69.85	4.87	0.83	0.20	0.06	0.02	0.01	0.00	0.00	0.00
Females, 2050 vs. 1900	83.84	49.07	21.29	7.36	2.93	1.41	0.76	0.42	0.24	0.14	0.23
Males, 2050 vs. 1900	75.84	46.56	20.14	6.09	1.86	0.66	0.27	0.12	0.06	0.03	0.05
Females vs. Males, 1900	49.07	46.56	2.41	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Females vs. Males, 1980	77.53	69.85	6.18	1.18	0.26	0.05	0.01	0.00	0.00	0.00	0.00
Females vs. Males, 2050	83.84	75.84	6.31	1.30	0.30	0.06	0.01	0.00	0.00	0.00	0.00

$$H_i = \frac{\int_0^{\omega} l(x) (-\ln l(x))^i / i! dx}{\int_0^{\omega} l(x) dx} \quad (36)$$

Note that  $H_1$  is the familiar expression for  $H$  given by Keyfitz (1985); it is clear from (35) that this expression for  $H$  is valid when  $\delta$  is small. (See Vaupel and Yashin (1985) for further discussion of this and of the logic of approaches based on finite differences vs. infinitesimal differentials.) Following the method used by Vaupel (1986), an alternative expression for  $H_i$  can be developed:

$$H_i = \frac{\int_0^{\omega} \mu(x) l(x) e(x) (-\ln l(x))^{i-1} / i! dx}{\int_0^{\omega} l(x) dx} \quad (37)$$

This expression, for  $H_1$ , is the expression for  $H$  used by Vaupel (1986) to analyze how change in age-specific mortality affects life expectancy.

Table 4 presents values of  $H_1$  through  $H_9$  for U.S. males and females at 1900, 1980, and 2050 mortality rates. Note how slowly the values of  $H_i$  fall off as  $i$  increases. It is not difficult to show that the sum of the  $H_i$ 's increases without limit as  $i$  increases, which is intuitively reasonable since if lives are saved over and



over again indefinitely, then life expectancy should grow with limit. That the values of  $H_i$  fall off as  $i$  increases is explained by the fact that persons whose lives are saved repeatedly are likely to be older persons who face high rates of deaths: if mortality rates were constant over age, the values of the  $H_i$ 's would all be the same (and equal to one) and if mortality rates declined with age, the values of the  $H_i$ 's would be increasing.

Table 4. Values of life expectancy at birth and of  $H_1, H_2, \dots, H_9$ , for U.S. females and males at 1900, 1980, and 2050 mortality rates.

Mortality regime	$e(0)$	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$	$H_7$	$H_8$	$H_9$
Females, 1900	49.07	0.480	0.205	0.111	0.075	0.058	0.047	0.040	0.034	0.028
Males, 1900	46.56	0.516	0.225	0.122	0.082	0.064	0.052	0.044	0.038	0.032
Females, 1980	77.53	0.155	0.071	0.048	0.037	0.031	0.026	0.023	0.019	0.016
Males, 1980	69.85	0.193	0.092	0.062	0.048	0.040	0.034	0.029	0.025	0.021
Females, 2050	83.84	0.144	0.069	0.048	0.038	0.031	0.026	0.021	0.017	0.012
Males, 2050	75.84	0.176	0.089	0.063	0.051	0.042	0.036	0.030	0.025	0.020

By multiplying the value of  $H_i$  by the prevailing life expectancy, the life years gained by saving a person's life the  $i$ 'th time can be calculated. Vaupel and Yashin (1985) explain the logic of this in detail; the basic idea is that if a random person's life is saved the  $i$ 'th time, then he or she can expect to live  $H_i$  times  $e(0)$  years in state  $i$ . These additional years of life expectancy represent the benefit of saving a life the  $i$ 'th time, not including the further benefit that might arise if the individual were saved again, the  $i+1$ -st time. Table 5 summarizes the effects of repeatedly averting death by presenting the average lifespans of people saved not at all, once and only once, twice and only twice, and so on up to nine times, in 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900, for U.S. females and for males.

The high values of  $H_1$  in 1900 imply that if every female's first death could be averted, 23.56 years would be added to female life expectancy, increasing life expectancy from 49.07 years to 72.63 years. Similarly, averting every male's first death would increase male life expectancy by 24.04 years, from 46.56 years to 70.60 years. It is intriguing to note that the actual increases in female and male life expectancy from 1900 to 1980 are roughly comparable to these gains: the progress achieved is equivalent to saving every female's life a bit more than once and every male's life a bit less than once.

Table 5. Average lifespans of people resuscitated 0, 1, 2,..., 9 times before ultimate death, for U.S. females and males at 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900 mortality rates.

Comparison	Average lifespans of those resuscitated:									
	0	1	2	3	4	5	6	7	8	9
Females, 1980 vs. 1900	49.07	72.63	82.67	88.10	91.79	94.63	96.95	98.90	100.55	101.93
Males, 1980 vs. 1900	46.56	70.60	81.09	86.76	90.60	93.56	95.99	98.05	99.81	101.30
Females, 2050 vs. 1980	77.53	89.53	95.03	98.75	101.66	104.07	106.12	107.86	109.33	110.55
Males, 2050 vs. 1980	69.85	83.32	89.73	94.08	97.43	100.19	102.54	104.56	106.29	107.74
Females, 2050 vs. 1900	83.84	95.88	101.68	105.73	108.93	111.57	113.75	115.51	116.89	117.94
Males, 2050 vs. 1900	75.84	89.19	95.96	100.77	104.60	107.80	110.50	112.77	114.66	116.17

At 1980 and 2050 rates, averting everyone's first death would be about half as beneficial: female life expectancy would increase about 12 years and male life expectancy by somewhat more than 13 years. Averting everyone's second death would add another 5 and a half years to female life expectancy and another 6 and a half years to male life expectancy, putting female life expectancy in 2050 at nearly 102 years and male life expectancy at nearly 96 years.

Each additional death averted adds fewer years to life expectancy, but the cumulative effect can be substantial. The average male facing 1980 death rates will survive to celebrate his 100th birthday if his life is saved five times. If a woman at 2050 rates could save her life nine times, she could expect to live to age 118.

### How Many Times Has Your Life Been Saved?

If mortality rates are reduced, some people will have their lives saved once, some twice, some many times before they finally die and other people will die at the same age they would have died before. Let  $n(x)$  be the average number of times people age  $x$  have had their lives saved:

$$n(x) = \sum_{i=0}^{\infty} i \pi_i(x) \quad , \quad (38)$$

where  $\pi_i(x)$  denotes the proportion of people age  $x$  who have had their lives saved  $i$  times, as given by (26). Substituting (26) and simplifying yields the surprising result that

$$n(x) = \Lambda(x) \quad . \quad (39)$$

Thus  $\Lambda(x)$ , the key variable in many of our formulas, can be interpreted not only as the cumulative intensity of lifesaving but also as the average number of times death has been averted.

The average number of times a newborn can expect to have his or her life saved before inevitable, final death is given by

$$\bar{n} = \int_0^{\omega} f'(x) n(x) dx \quad . \quad (40)$$

This value summarizes the difference between two mortality regimes and thus might be used to measure the distance between two levels of mortality. Change in life expectancy is the measure usually used;  $\bar{n}$  represents an alternative that can be employed when one mortality trajectory is at least as low as a second trajectory, at all ages.

Table 6 presents values of  $n(x)$  at selected ages as well as  $\bar{n}$ , for nine different pairs of mortality regimes. On average, the older a person gets, the more times his or her life has been saved: the average 100-year-old female at 2050 vs. 1900 rates will have had her life saved six times. At birth, a newborn girl at 2050 vs. 1900 rates can expect to be reprieved from death more than 2.7 times, whereas a newborn boy can expect only 1.7 resuscitations: that extra life measures the greater progress made in lowering female mortality. The gap between male and female life chances can be measured by comparing the two regimes: to achieve female life expectancy in 1900, the average male would have to be resuscitated 0.11 times; in 1980 and 2050, the required average number of resuscitations is six times higher.

When  $\delta(x)$  is constant, the formula for  $\bar{n}$  collapses to the remarkably simple result

$$\bar{n} = \frac{\delta}{1 - \delta} \quad . \quad (41)$$

Thus, if mortality is cut in half at all ages, a newborn can expect to be reprieved from death once; if mortality is cut to a quarter of its original level, the average newborn will be resuscitated three times.

That the average newborn can expect to be reprieved from death  $\bar{n}$  times does not mean that all newborns will be reprieved  $\bar{n}$  times. On the contrary, some members of the birth cohort will benefit many times from lifesaving and others will

Table 6. Average number of resuscitations for those alive at selected ages ( $n(a)$ ) and expected number of resuscitations in a lifetime ( $\bar{n}$ ), for U.S. females and males at 1980 vs. 1900, 2050 vs. 1980, and 2050 vs. 1900 mortality rates and for females vs. males at 1900, 1980, and 2050 rates.

Comparison	$n(10)$	$n(30)$	$n(50)$	$n(70)$	$n(90)$	$n(100)$	$\bar{n}$
Females, 1980 vs. 1900	0.21	0.32	0.49	0.86	2.40	4.18	1.61
Males, 1980 vs. 1900	0.24	0.33	0.48	0.71	1.77	3.07	0.91
Females, 2050 vs. 1980	0.01	0.01	0.03	0.10	0.75	1.82	0.76
Males, 2050 vs. 1980	0.01	0.02	0.04	0.20	0.96	2.17	0.57
Females, 2050 vs. 1900	0.22	0.33	0.51	0.96	3.15	5.99	2.76
Males, 2050 vs. 1900	0.25	0.34	0.53	0.91	2.72	5.23	1.70
Females vs. Males, 1900	.03	.04	.05	.12	.40	.43	0.11
Females vs. Males, 1980	.00	.02	.06	.27	1.04	1.54	0.63
Females vs. Males, 2050	.00	.02	.04	.17	.83	1.19	0.66

not benefit at all. Consider the simple case where mortality rates are cut in half at all ages, such that  $\bar{n}$  is one. At the moment death would have occurred, half of the individuals are reprieved--and the other half die as before. This same bifurcation occurs among those resuscitated once, then among those resuscitated twice, and so on. Thus, half of the cohort do not benefit from the lifesaving, a quarter are reprieved once, an eighth are reprieved twice, and an eighth are reprieved more than twice. The quarter who are reprieved once and the eighth who are reprieved twice each account for a quarter of the total reprieves: this means that the fortunate eighth who are reprieved more than twice account for fully half of the total reprieves. There is, in short, a concentration of resuscitation such that an eighth of the people get half the benefits. This concentration arises even though, indeed because, the process of lifesaving is completely democratic--everyone has an equal chance, at birth, of having his or her life saved. As in many other situations, an equitable process leads to a very unequal outcome. (For further discussion of concentrations in populations, see Goodwin and Vaupel (1985).)

## Extensions

In two companion papers, Vaupel (1986) and Vaupel and Yashin (1985), we initiate some lines of analysis that, combined with the approach presented here, could lead to some additional results of interest:

- (1) Vaupel (1986) analyzes how change in age-specific mortality affects life expectancy. His approach, which is based on methods of differential calculus, is useful when small changes in age-specific mortality are being considered. When age-specific mortality changes substantially, the approaches proposed by Pollard (1982) and in a United Nations study (1982) are appropriate. An alternative decomposition of the effects of age-specific mortality change on life expectancy could be based on the notion of repeated resuscitation.
- (2) The lifesaving model developed here assumes that all individuals face the same chances of death and that the resuscitated have the same life chances as those who did not have to be saved from death. It seems clear, however, that individuals differ in their frailty (Vaupel, Manton, and Stallard 1979) and that this heterogeneity will alter the impact of lifesaving on life table statistics. The results in this current paper could be extended along the lines we discuss in Vaupel (1986) and Vaupel and Yashin (1985).
- (3) In Vaupel and Yashin (1985) we develop a model, which we call the second-chance model, that permits individuals' lives to be saved once but only once. A comparison of that model with the repeated resuscitation model could yield some stimulating insights. For instance, is it better to save everyone's life once or to save lives once on average? The second-chance model may be useful in evaluating the effects of heterogeneity in life chances, since the mortality rates of those who are resuscitated can be set at a higher level than the rates for those whose lives have not been saved.

The illustrations presented in this paper have all concerned human mortality, but studies of other kinds of population attrition, pertaining to morbidity, marriage, divorce, abortion, unemployment, animal stocks, pest control, equipment failure, and so on, might also benefit from application of the notion of lifesaving and the repeated-resuscitation model. Indeed, in some of these areas the model may be especially relevant and useful because lifesaving may be under the direct and effective control of a decision maker. Consider, for instance, the case of equipment that can be repaired (and thus resuscitated) if it fails. The model could be used to investigate the optimal number of times the equipment should be repaired, the expected number of times the equipment will be repaired before ulti-

mate scraping, and the concentration of repair among the equipment population. In an example analogous to that given above, it could be shown that an eighth of the equipment requires half the repairs--even though there are no lemons, all the equipment having equal chances of failure.

### **Conclusion**

The notion of lifesaving can be combined, as this paper has illustrated, with standard methods of demographic analysis to yield insights about how progress against mortality influences patterns of survivorship, the density distribution of deaths, and life expectancy. Furthermore, thinking about progress against mortality in terms of lifesaving--e.g., the number of times a newborn's life will be saved if mortality is reduced from one level to another--helps clarify the nature and significance of this progress.

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