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Steady State Analysis of the Finnish Forest Sector

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STEADY STATE ANALYSIS OF THE FINNISH
FOREST SECTOR

Markku Kallio
Margareta Soismaa

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FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade of wood products. Such issues include the development of the global economy and population, new wood products and substitution for wood products, future supply of roundwood and alternative fiber sources, technology development for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequences of future expectations and assumptions concerning such substantive issues.

The research program of the Project includes an aggregated analysis of long-term development of international trade in wood products, and thereby analysis of the development of wood resources, forest industrial production and demand in different world regions. The other main research activity is a detailed analysis of the forest sector in industrial countries. Research on these mutually supporting topics is carried out simultaneously in collaboration between IIASA and the collaborating institutions of the Project.

This paper is a specific study of the Finnish forest sector. Its goal is to analyze one of the main cost factors, the wood cost, and the effect of this internally priced factor on the competitiveness of the Finnish forest industry.

Markku Kallio
Project Leader
Forest Sector Project

ABSTRACT

During the recent years the total cost of round wood for the Finnish forest industry has been in the order of US$ 1.5 billion annually. The share of stumpage price represents roughly one half whereas harvesting, transportation etc account for the rest. The purpose of this study is to investigate long term equilibrium prices for wood (and thereby total round wood costs) under various conditions of world market of wood products.

In the first part of this paper a (discrete time) dynamic linear model for the forest sector is discussed. The steady state version of it is analyzed in more detail. An application of the steady state forestry model is carried out for the Finnish forests. As a result, alternative sustained yield solutions for the Finnish forests are obtained.

In the analysis of the second part, a steady state sectorial model is adopted to carry out a Stackelberg equilibrium analysis for the round wood market. Further elaboration appeared necessary until the steady state model became suitable for this game theoretic analysis. This elaboration involves definitions of objective functions of the key parties (the forestry and the industry) in the forest sector game. A demand function of constant price elasticity is adopted for wood products.

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STEADY STATE ANALYSIS OF THE FINNISH FOREST SECTOR

Markku Kallio and Margareta Soismaa

1 Introduction

During the last few years a growing research effort has been directed towards (renewable) natural resources. Since the prosperity of many nations is dependent on a sensible exploitation of these resources the significance of studies dealing with such problems becomes great. For the Finnish economy forests represent the most important national resource. Not only as such but because an entire line of production ranging from pulp and sawn goods to paper, furniture and prefabricated houses, is based on wood not to mention industry producing machinery for forestry and wood processing. This also emphasizes the importance of the forest sector, which includes both forestry and forest based industries, for employment and foreign trade.

In the past the total cost of round wood for the Finnish forest industry has been of the order of US$ 1.5 billion annually. The share of stumpage price represents roughly one half whereas harvesting, transportation, etc account for the rest. The purpose of this study is to investigate long range equilibrium prices for wood (and thereby total round wood cost) under various conditions for world market of wood products.

In Section 2 we will present a dynamic linear model for the entire forest sector. In Section 3 we take forestry separately and determine the optimal harvesting policies in a steady state. Section 4 deals with a steady state model for the forest industries. In Section 5 we combine these two parts and formulate a steady state model for the forest sector. In Section 6 we supplement the model of Section 5 to make it applicable for solving the long range equilibrium wood prices as solution to a Stackelberg game. In Section 7 we present numerical results from experiments with Finnish data. Finally, Section 8 stands for summary and conclusions.

2 A Dynamic Linear Model for the Forestry and Wood Processing Industry

We shall consider first the integrated and dynamic system of wood supply and wood processing; ie forestry and forest industry. The model has been adopted from Kallio, Propoi, and Seppala /2/. The discussion begins with the forestry part describing the growth of the forest given harvesting and planting activities, as well as land availability over time. The wood processing part consists of an input-output model describing the production process as well as of production capacity and financial resource considerations. Each part is a discrete time linear model describing its object over a chosen time interval.

2.1 Forestry

Let w(t) be a vector determinig the number of various tree species (say pine, spruce and birch) in different age categories at the beginning of time period t. We define a square transition (growth) matrix Q so that Qw(t) is the number of trees at the beginning of period t+1 given that nothing is harvested or planted. Thus, matrix Q describes aging and natural death of the trees. Let p(t) and h(t) be vectors for levels of different kinds of planting and harvesting activities, respectively (eg planting of different species, terminal harvesting, thinning, etc), and let the matrices P and H be defined so that Pp(t) and -Hh(t) are the incremental increase in the tree quantinty caused by the planting and harvesting activities. Then, for the state vector w(t) of the number of trees in different age categories we have the following equation:

(2.1) $$w(t+1) = Qw(t) + Pp(t) - Hh(t) \quad .$$

Planting is restricted through land availability. We may formulate the land constraint so that the total stem volume of trees in forests cannot exceed a given volume L(t) during t. Thus, if W is a vector of stem volume per tree for different species in various age groups, then the land availability restriction may be stated as

(2.2) $$Ww(t) \leq L(t) \quad .$$

Given the level of harvesting activity h(t), there is a minimum requirement for the planting activity p(t) (required by the law, for instance) as follows:

(2.3) $$p(t) \geq Nh(t) \quad ,$$

where N is the matrix transforming the level of harvesting activities to planting requirements.

In this simple formulation we shall leave out other restrictions, such as harvesting labor and capacity. Finally, the wood supply y(t), given the level of harvesting activities h(t), is given for period t as

(2.4) $$y(t) = SHh(t) \quad .$$

Here the matrix $S = (S_{ij})$ transforms a tree of a certain species and age combination j into a volume of type i of timber assortment (eg pine log, spruce pulpwood, etc).

2.2 Wood Processing Industry

For the industrial side, let x(t) be the vector of production activities for period t (such as the production of sawn goods, panels, pulp, paper, and converted wood products), and let U' be the matrix of wood usage per unit of production activity. The wood demand for period t is then given by U'x(t). It cannot exceed wood supply y(t):

$$(2.5) \qquad y(t) \geq U'x(t)$$

Note that the matrix U' may also have negative elements. For instance, sawmill activity consumes logs but produces pulpwood as a residual.

Let A be an input-output matrix so that $(I - A)x(t)$ is the vector of net production. If $D(t)$ is the corresponding (maximum) external demand, we require

$$(2.6) \qquad (I - A)x(t) \leq D(t) \quad .$$

Production is restricted by the capacity $c(t)$ available:

$$(2.7) \qquad x(t) \leq c(t) \quad .$$

The vector $c(t)$ in turn has to satisfy the state equation

$$(2.8) \qquad c(t+1) = (I - g)c(t) + v(t) \quad ,$$

where g is a diagonal matrix accounting for (physical) depreciation and $v(t)$ is the increment from investments during period t. The vector $v(t)$ of investment activities is restricted through financial considerations. To specify this, let $m(t)$ be the state variable for cash at the beginning of period t. Let $G(t)$ be the vector of sales revenue less direct production costs per unit of production including, for instance, wood, energy, and direct labor costs. Let $F(t)$ be the vector of monetary fixed costs per unit of capacity, let $l(t)$ be the amount of external financing employed by the industry at the beginning of period t, let s be the interest rate for external financing per period, let $l^+(t)$ be new loans taken during period t, let $l^-(t)$ be loan repayments during t, and let $E(t)$ be the vector of cash expenditure per unit of increase in the production capacity. Then, the state equation for cash may be written as

$$(2.9) \qquad m(t+1) = m(t) + G(t)x(t) - F(t)c(t)$$

$$-sl(t) - l^-(t) + l^+(t) - E(t)v(t) \quad .$$

Finally, for the industrial model, we may write the state equation for external financing as follows:

w(t+1)	w(t)	p(t)	h(t)	y(t)	x(t)	c(t)	v(t)	m(t)	l(t)	$l^+(t)$	$l^-(t)$	c(t+1)	m(t+1)	l(t+1)	
I	-Q	-P	H												= 0
		I	-N												≥ 0
	W														≤ L
			-SH	I											= 0
				I	-U′										≥ 0
					I - A										≤ D
					I	-I									≤ 0
						-(I - g)	-I					I			= 0
					-G	F	E	-1	s	-1	1		1		= 0
									-1	-1	1			1	= 0

Figure 1. Constraint matrix of the forest sector model for period t.

$$(2.10) \qquad l(t+1) = l(t) - l^-(t) + l^+(t) \quad .$$

Figure 1 presents the structure of the constraint matrix of the forest sector model for period t.

3 Sustained Yield in Forestry

In the previous section we presented a dynamic linear programming model encompassing both forestry and forest based industries. In this section we focus our attention solely on forestry. We present forestry in a steady state by assuming that one period follows another one without changes. We shall investigate alternative sustained and efficient timber yields in various timber assortments. We also present an application to the Finnish forestry.

3.1 The Steady State Formulation

In Section 2.1 we presented a general dynamic formulation for forestry. In this section we deal with a steady state case of this model.

We consider a forest land with a single tree species and with uniform soil, climate, etc conditions. We assign the trees to age groups a, for $a = 1, 2, \ldots, N$. Let d be a time interval; eg 5 years. A tree belongs to age group a if its age is in the interval $[(a-1)d, ad)$ for all $a < N$. Trees which have an age of at least $(N-1)d$ belong to age group N. We consider a discrete time steady state formulation of the forest, where each time period is also an interval of d. Let p be the number of trees entering the first age group during each period (eg through planting or natural regeneration), and let w(a) be the number of trees in age group a at the beginning of each time period, for $1 \leq a \leq N$ (cf (2.1)). Let h(a) be the number of trees harvested during each period from age group a. In this case, we assume that the harvesting activities equal the number of trees harvested from each age group during each period. We denote by Q_a the ratio of trees proceeding from age group a to group a+1 in one period given that no harvesting occurs. Without loss of generality we assume $0 \leq Q_a \leq 1$, for all a.

Factors $(1-Q_a)$ account for natural death of trees, forest fires, etc as well as for thinning of forests in age group a. The state equations for forestry in a steady state may then be written as follows (cf (2.1)):

$$(3.1) \qquad w(1) = p \quad ,$$

$$(3.2) \qquad w(a+1) = Q_a w(a) - h(a) \quad , \quad 1 \leq a \leq N \quad ,$$

where we define $w(N+1) = 0$.

The land constraint prevents excessive planting (cf (2.2)). Let W_a be the amount of land consumed by each tree in age group a, $1 \leq a \leq N$, and let L be the total amount of land available in the forests. Alternatively, the space limitations may be taken into account denoting by W_a the volume of wood per tree in age group a and by L the total possible volume of wood in the forests. In either case the land constraint is given as

$$(3.3) \qquad \sum_{a=1}^{N} W_a w(a) \leq L \quad .$$

As a performance index for forestry we consider the physical wood supply. (Experience shows that when we maximize the physical wood supply we usually get a policy which also meets other important requirements, such as preserving the watershed.) The timber assortments vary in value (eg log, pulpwood, fuel wood). Let j (=1, 2, ...) refer to different timber assortments. Accordingly, let e_{aj} be the yield (in m^3/tree) of timber assortment j when a tree in age group a is harvested, and let g_{aj} be the yield per tree in age group a resulting from thinning activities. As stated earlier, our objective is to find an efficient timber yield using the yields of timber assortments as criteria. Let e_a and g_a be convex combinations (weighted sums) of the coefficients e_{aj} and g_{aj}, respectively. The objective is to maximize the weighted sum of the yields of various timber assortments and it is given as

$$(3.4) \qquad \sum_{a=1}^{N} (e_a h(a) + g_a w(a)) \quad .$$

The weights to be used may be proportional to the market prices of the timber assortments. Also other weights may be considered for studying efficient yields (see Section 3.2 below). The forestry problem, denoted by (F), is to find nonnegative scalars h(a) and w(a), for each a, which maximize (3.4) subject to (3.1)-(3.3). The following result is used to derive an optimal solution to this linear program:

Proposition: For an optimal solution of the forestry problem (F) there is an age group A such that h(a) = 0, for all a $\neq$ A, and w(a) = 0, for all a > A.

Thus in the optimal harvesting schedule, all trees are harvested, clearcut (besides thinning activities) if and only if they reach age group A. Therefore, there are no trees in age groups higher than A. Problem (F) may then be solved, for instance, checking all alternative harvesting policies of this type. - For a proof of the Proposition, see Appendix 1.

We consider now a particular policy a = A where trees are harvested in an age group A. Then, according to (3.2),

$$(3.5) \qquad w(a) = \begin{cases} p \prod_{i<a} Q_i & \text{for } a \leq A \\ 0 & \text{for } a > A. \end{cases}$$

For the corresponding level of planting p_A the land constraint (3.3) yields:

$$(3.6) \qquad p_A = L / \left(\sum_{a=1}^{A} W_a \prod_{i<a} Q_i \right) \quad .$$

The number of trees harvested, when policy A is applied, is given as

$$(3.7) \qquad h(A) = Q_A \; w(A) \quad .$$

The efficient yield of timber assortment j from clearcutting when policy A is applied is given as

(3.8) $$e_{Aj}\ h(A) \quad .$$

As for cutting and thinning, the efficient yield of timber assortment j under policy A is

(3.9) $$\sum_{a=1}^{A} g_{aj}\ w(a) \quad .$$

3.2 Application to the Finnish Forestry

We will now apply this approach to the forestry in Finland. Let the age group interval d be 5 years and N = 21 (so that the oldest group includes trees of at least 100 years old). We consider two timber assortments: pulpwood (j=1) and log (j=2).

Table 1 gives estimates for the transition probabilities Q_a, the average volume of pulpwood and log per tree in age group a e_{a1} and e_{a2}, respectively, as well as the total volume W_a. We assume that all losses indicated by the Q_a coefficients for age groups less than 20 are due to thinning. Based on this, the yield coefficients g_{aj} can be given as

(3.10) $$g_{aj} = (1-Q_a)e_{aj} \quad ,$$

for $4 \leq a < 20$. We assume $g_{aj} = 0$ for each j, for $a \geq 20$. The land constraint (3.3) requires that the total volume of log and pulpwood cannot exceed an amount of L=1700 million m^3, which is around ten percent above the actual current level in Finland. According to the transition coefficients, 5.6 trees have to be planted for each grown tree harvested when policies A = 14, 15, ..., 21 are applied. This number is roughly what is enforced by the Finnish law today.

Figure 2 shows the annual yield of log and pulpwood when harvesting policies A = 12, 13, ..., 21 are applied. We may note that alternatives A = 17, 18, ..., 21 are dominated by other alternatives; ie there is another alternative whose yield is better for both of the two timber assortments. The optimal alternative depends on the weighting of log and pulpwood. If the weight for log is at least 150 percent larger than that for pulpwood, then A = 16 is optimal; ie a tree gets harvested when it grows 75 to 80 years old. If this percentage is 100 (which roughly corresponds to the current price levels of log and pulpwood in Finland) then the alternatives A = 14 and A = 15 are about equally good; ie trees in the age interval 65 to 75 should be harvested. When the weight for log drops to only 75 percent above that for pulpwood, the harvesting age falls to 60 to 65 years.

Table 1. Transition coefficients Q_a, volume W_a, pulpwood yield e_{a1} and g_{a1}, and log yield e_{a2}. Yield g_{a2} equals .0 for all age groups except for a=13 for which g_{a2} = .003 (Volumes in m^3/tree)

a	Q_a	W_a	e_{a1}	e_{a2}	g_{a1}
1	.68	.0	.0	.0	.0
2	.93	.0	.0	.0	.0
3	.90	.001	.001	.0	.0
4	.80	.006	.006	.0	.001
5	.80	.014	.014	.0	.003
6	.82	.026	.026	.0	.005
7	.90	.041	.041	.0	.004
8	.93	.061	.061	.0	.004
9	.93	.085	.085	.0	.006
10	.93	.114	.114	.0	.008
11	.97	.146	.146	.0	.004
12	.97	.182	.182	.0	.005
13	.97	.222	.138	.084	.004
14	1.	.263	.113	.150	.0
15	1.	.308	.102	.206	.0
16	1.	.353	.081	.272	.0
17	1.	.399	.076	.323	.0
18	1.	.446	.071	.375	.0
19	1.	.494	.064	.430	.0
20	.99	.543	.060	.483	.0
21	.95	.600	.060	.540	.0

The yield along the line segment between the corner points in Figure 2 may be obtained when two policies are combined. Logs may also be used as pulpwood.

This has been illustrated for A = 21 by points along the broken line of Figure 2. Note that each such point is inferior to the efficient frontier, and the same is true for any other policy alternative A. Thus, in the stationary state logs should not be used as pulpwood regardless of the price ratio of log and pulpwood. (In a transition period this of course may not hold.)

Figure 2. Alternative yield of log and pulpwood.

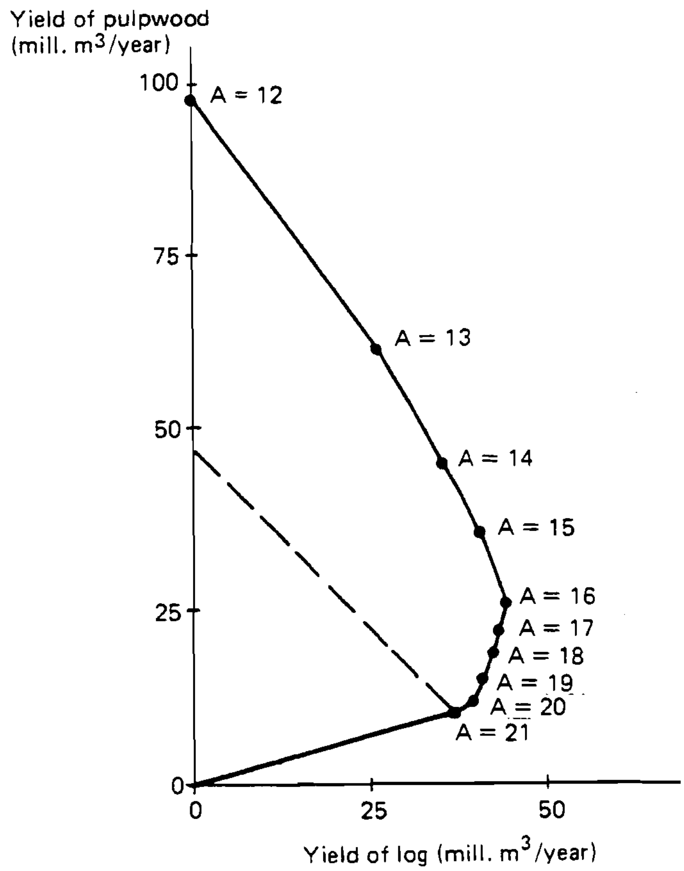


Table 2 summarizes the alternatives A = 12, 13, ..., 16. It shows, for each policy alternative A, the yields of log and pulpwood separately from the harvesting and the thinning activities when the total volume L of forests is assumed to be 1700 mill. m^3. Also the number of trees to be harvested and planted annually is shown in Table 2.

The age distribution of trees resulting from policy alternatives A = 13, ..., 16 has been illustarted in Figure 3. For comparison, the estimated age distribution in 1976 adjusted to the same total volume of forests has been shown in Figure

3. In Figure 4 we have presented the distribution of the volume of trees in different age groups for policies A=13, 14, 15, and 16. The estimated distribution of the volume for the year 1976 has also been presented.

Table 2. Yield by timber assortments, trees harvested and trees planted for harvesting policies A = 12, ..., 16.

Harvesting policy A	12	13	14	15	16
Log yield, mill. m^3/a					
Harvesting	0	24.1	35.0	39.8	43.9
Thinning	0	3.7	2.7	2.3	2.0
Total	0	27.8	37.7	42.1	45.9
Pulpwood yield, mill. m^3/a					
Harvesting	76.2	39.5	26.5	19.7	13.0
Thinning	22.7	24.0	19.7	16.3	13.6
Total	98.9	63.5	46.2	36.0	26.6
Total yield, mill. m^3/a	98.9	91.3	83.9	78.1	72.5
Harvesting, mill. trees/a	420	290	230	190	160
Planting, mill. trees/a	2060	1620	1320	1090	910

Figure 3. Age distribution of trees for policies A = 13, ..., and comparison with the situation in 1976.

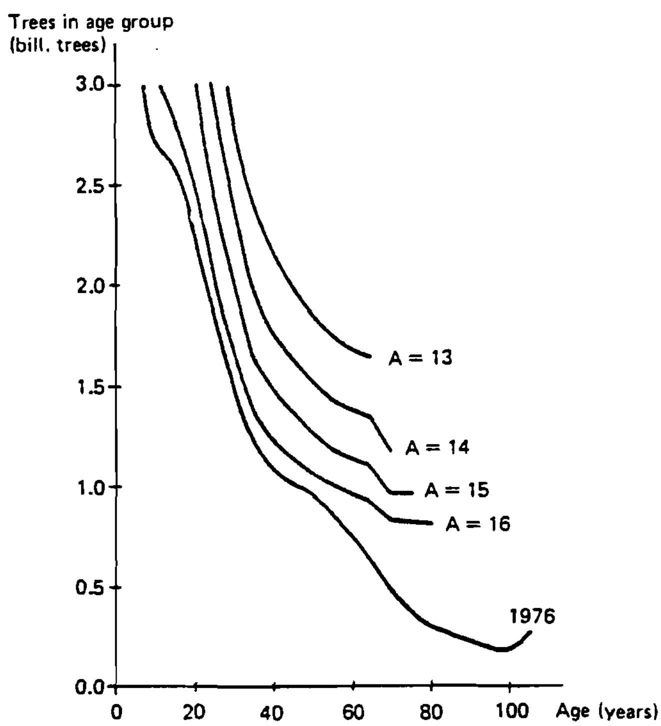
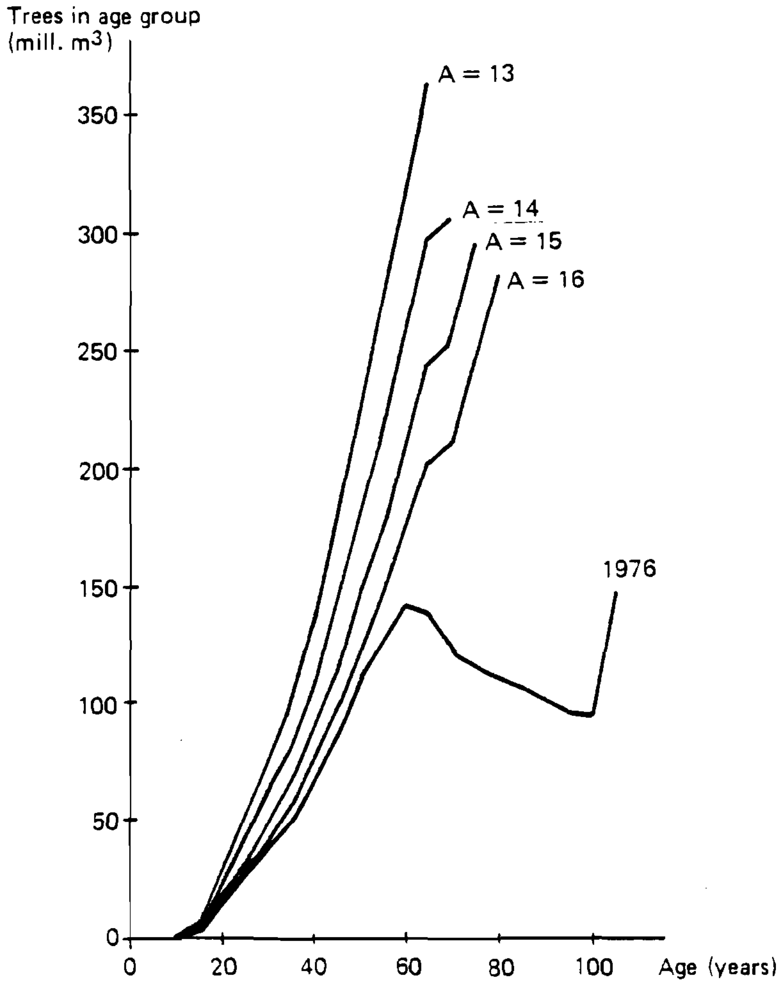


Figure 4. The volume distribution of trees in age groups for policies A = 13, 14, 15, and 16 as compared to the situation of 1976.



4 A Steady State Model of the Forest Industries

In this section we shall consider the wood processing part of the model of Section 2 in a steady state.

Suppressing the time index t in the industrial part of the model, equation (2.5) yields

(4.1) $$U'x \leq y \quad ,$$

ie industrial usage of wood U'x cannot exceed wood supply y.

Equation (2.6) requires that net production (I - A)x supplied to the external market cannot exceed demand D:

(4.2) $$(I - A)x \leq D.$$

As in the dynamic version (2.7), gross production is limited by capacity c

(4.3) $$x \leq c \quad .$$

The state equation (2.8) for capacity yields

(4.4) $$gc = v \quad ,$$

ie investments v equal (physical) depreciation gc. The state equation (2.10) for external financing is rewritten as

(4.5) $$l^{-} = l^{+} \quad ,$$

in other words, the level of external financing remains constant in the steady state formulation.

Taking into account (4.4) and (4.5) the modification of (2.9) results in the following formulation

(4.6) $$Gx - (F + Eg)c - sl = 0$$

Equation (4.6) states that the net income from sales equals the expenditures caused by capacity (fixed costs and depreciation) plus external financing (interest payments). Alternatively we may replace equality in (4.6) by an inequality. The slack can then be interpreted as a constant flow out from the forest sector.

It is obvious that in the optimal solution (4.3) holds as an equality:

(4.7) $$x = c \quad .$$

We define a vector d for the external demand which equals net production. Solving x from this, yields

(4.8) $$U'(I - A)^{-1} d \leq y \quad .$$

In summary, for the steady state solution we have to find d which satisfies (4.8),

(4.9) $$(G - F - Eg)(I - A)^{-1} d \geq 0$$

and

(4.10) $$0 \leq d \leq D \quad .$$

5 A Steady State Model of the Forest Sector

Above we have presented two steady state models one, for forestry and another for wood processing industries. In this section we shall merge these two parts to obtain a steady state model for the entire forest sector.

Efficient yields of pulpwood and log are shown by Figure 2, in which the feasible region of yields can be defined by a set of linear inequalities:

$$R\,y \leq s \quad ,$$

where y is a vector of m components signifying the different timber assortments, R is a matrix and s a vector. For the two-dimensional case of Figure 2, the components of R and s can be obtained immediately.

Thus, for U'x, the industrial use of wood, we require

$$R\,U'x \leq s \quad ,$$

or

$$(5.1) \qquad R\,U'(I - A)^{-1}\, d \leq s.$$

The steady state solution d for the entire sector is then one which satisfies (4.9), (4.10) and (5.1).

So far we have not included into the model any objective functions. For forestry we might choose to maximize stumpage earnings; ie the income from selling wood to industry less the production costs for that wood (eg harvesting and transportation costs). As for industry, industrial profit, the left-hand side of equation (4.9) offers one possible objective function to be maximized. The sum of these two could constitute a joint objective function (the joint profit) for the entire forest sector. We shall discuss this subject further in Section 6.1.

6 A Stackelberg Game

In this section we shall discuss a specification of the steady state model to be applied for timber market analysis. The model will be augmented with objective functions both for forestry and wood processing industry. Furthermore, the demand for final products is represented by a demand function of constant price elasticity. The round wood market is viewed as a Stackelberg game.

It is apparent that the game situation in the forest sector involves two parties: forestry and forest industry. So far this bipartition has been revealed by separate models for each party. These models are interconnected through the amount of round wood supplied by forestry to the industry and through the prices of round wood.

The market mechanism which determines (round) wood prices may be described as follows: Given the prices and the availability of different timber assortments (at these prices) the industry chooses the quantity it will buy by maximizing its profit; the problem for forestry is to choose prices to maximize its profit (given the resulting wood demand for that price).

The decision process described above is called a Stackelberg game /4/. The party making the first decision (on prices) is called the leader of the game and the other party the follower. In our application, forestry acts as the leader and the industry as the follower. We assume that both the leader and the follower are **profit maximizers** and that they both have **perfect information** on the game (eg on profit functions, supply and demand).

The compexity of the game arises from the fact that the profits of both parties depend on raw wood prices. The revenues of forestry is determined by the price of wood and the quantity sold. In addition, the production cost for wood (eg harvesting and transportation costs) influence the profit of forestry. For the industry, the price of wood influences the cost of production. The sales price of an industrial product influences its demand.

At the solution of the game, ie at the Stackelberg equilibrium, prices for timber assortments are at a level which maximizes forestry's profit taking into account the effect of this price level on wood demand.

6.1 The Profit Functions

In order to solve the (Stackelberg) equilibrium prices we shall append to the steady state model of Section 5 profit functions for both parties.

Let $p = (p_i)$ be the vector of unit prices for industrial products i on the international market, let vector $c = (c_i)$ stand for the costs of one unit of production including labor, energy and fixed costs, depreciation, and real interest on total invested capital but excluding wood cost. Let z be the vector of wood prices for the different timber assortments. Denote

$$(6.1) \qquad U = U'(I - A)^{-1}$$

as the vector of timber assortments required for one unit of (industrial) production. **Industrial profit,** denoted by P_I, is given by

$$(6.2) \qquad P_I = (p - c - zU)d \quad ,$$

where vector d stands for the volume of export.

As for forestry, denote by e the unit production cost for wood. **Forestry profit,** denoted by P_F, is given by

(6.3) $$P_F = (z - e)y \quad ,$$

where y is the quantity of wood sold to the industry.

6.2 Demand Functions and Optimal Prices for Wood Products

In Section 5, we assumed that the external demand for wood products is limited by an (exogenous) upper bound. However, for the Stackelberg analysis it is convenient to use a demand function with constant price elasticity

(6.4) $$d_i = k_i \, p_i^{-b_i} \quad ,$$

(for each wood product i) where p_i is the price, k_i is a constant, and $-b_i$ is the price elasticity of demand. We may assume that b_i is greater than 1.

Denote by $\bar{p}_i$ the world market price which results in the (reference level) of demand $\bar{d}_i$. For example, if $\bar{d}_i$ is the current (external) demand, then $\bar{p}_i$ shall refer to the current price. Using $\bar{p}_i$ and $\bar{d}_i$ we solve for k_i. Substituting into (6.4) yields

(6.5) $$d_i/\bar{d}_i = (p_i/\bar{p}_i)^{-b_i} \quad .$$

Inserting $d = (d_i)$ from (6.5) into (6.2), we can solve the (profit maximizing) optimal price p_i^* for wood products. As a result we have

(6.6) $$p_i^* = (b_i/(b_i-1))(c_i + zU) \quad .$$

6.3 The Profit Maximization Problem for Forestry

In (6.6) we obtain the optimal price p_i^* as a function of wood price z; in other words, $p_i^* = p_i^*(z)$. Thus, external (optimal) demand d_i is actually a function of wood price z. We shall denote the vector of optimal demand quantities as a function of z by d(z). The wood usage y = y(z) corresponding to the optimal wood product prices is then given as a function of wood price:

(6.7) $$y(z) = U' \, (I - A)^{-1} \, d(z)$$

According to (5.1), the wood availability from forests restricts wood consumption as follows:

(6.8) $$R \, U' \, (I - A)^{-1} \, d(z) \leq s \; .$$

We have combined the two models, one for forestry and another one for industry, to yield the following optimization problem for forestry:

(6.9) $$\max_{z} \; P_F(z) = (z - e) \; y(z)$$

subject to

(6.10) $$R \; y(z) \leq s \; .$$

The forestry profit maximizing wood price vector, denoted by z^*, is the (Stackelberg) equilibrium price.

7 Equilibrium Solutions for Finland

In this section we shall present numerical results for the Stackelberg game with Finnish data. We will carry out the numercal tests using a model dealing with two timber assortments (log and pulpwood) and with seven wood products: sawn goods, panels, other mechanical wood products, mechanical pulp, chemical pulp, paper, and converted paper products.

For the forestry sector we employ the alternative sustained yield solutions derived in Section 3. The set of sustained yield solutions of Figure 3 is used to define the constraints (6.10) defining the convex polyhedral set of feasible round wood yield.

For the industrial model, we assume demand functions with price elasticity coefficients b_i=b being equal for each product. According to the representatives of the Finnish forest industry, a reasonable assumption concerning the value of b is the range between 10 and 30. However, sensitivity analysis shall be presented for the whole range of $1 < b \leq \infty$.

Another highly sensitive and uncertain figure in the analysis is the reference level p_i of the world market price. For sensitivity analysis, three price scenarios were constructed for each forest product. Scenario 1: an **optimistic** world market price is defined as total production cost in Finland (including wood cost at present prices and a ten percent real interest on total invested capital). Scenario 3: a **pessimistic** price is defined reflecting such production costs for the major future suppliers (such as North American and Latin American producers) in the world market /1/. Scenario 2, a more **likely** scenario, is the average of the two above. According to our data, the price in Scenario 1 is higher than in Scenario 3 for each wood product separately.

7.1 The Single Product - Single Timber Assortment Case

For qualitative analysis of the model we shall first study the case of a single timber assortment and a single product. In this case, the equilibrium can actually be solved analytically.

Depending on the value of b the results shall be studied in two cases. We consider first the case when b is small and when forest land is not fully exploited. To solve the equilibrium wood price z^* we maximize forestry profit as defined in Section 6.3. Taking into account (6.5) and (6.6) and omitting constants we have

$$(7.1) \qquad P_F = (z-e)\bar{d}\ \bar{p}^b(b/(b-1))^{-b}(c+z)^{-b}$$

$$= (z-e)\ (c+z)^{-b}$$

The equilibrium wood price z^* from (7.1) is

$$(7.2) \qquad z^* = (c+be)/(b-1)$$

Notice that z^* is independent of the world market reference price $\bar{p}$. It is a decreasing function of b, which asymptotically approaches wood production cost e (harvesting, transportation, etc) as b increases.

Inserting z^* into (7.1) the maximum forestry profit is

(7.3) $$P_F = ((c+e)/(b-1))[(b/(b-1))^2((c+e)/\bar{p})]^{-b}\bar{d}$$

As for industrial profit given by (6.2), the following formula results

(7.4) $$P_I = (p-c-z^*)d = (b/(b-1))P_F$$

Along with b, forest utilization increases until the total forest land area is exploited. In this second case, when forest land is fully exploited, we solve the equilibrium wood price z^* assuming that the demand for round wood equals the maximum supply. The maximum production is denoted by d^*. From (6.5) and (6.6) we get

(7.5) $$d^* = \bar{d}(b/(b-1))^{-b}((c+z)/\bar{p})^{-b}$$

Solving the equilibrium wood price z^* from (7.5) results in

(7.6) $$z^* = (\bar{d}/d^*)^{1/b}((b-1)/b)\bar{p} - c$$

In this case, the equilibrium price z^* is a concave function of b which asymptotically approaches ($\bar{p}$-c) (the unit profit when wood cost is omitted) as b increases.

Inserting (7.6) into (6.6) yields the optimal wood product price

(7.7) $$p^* = \bar{p}(\bar{d}/d^*)^{1/b}$$

which asymptotically approaches $\bar{p}$ (the world market price) as b approaches infinity.

Using (7.4), (7.6), and (7.7) the industrial profit is defined as

(7.8) $$P_I = (1/b)(\bar{d}/d^*)^{1/b}\ \bar{p}d^*$$

As b increases the equilibrium price z^* asymptotically approaches a level absorbing all profit of the forest sector into wood price.

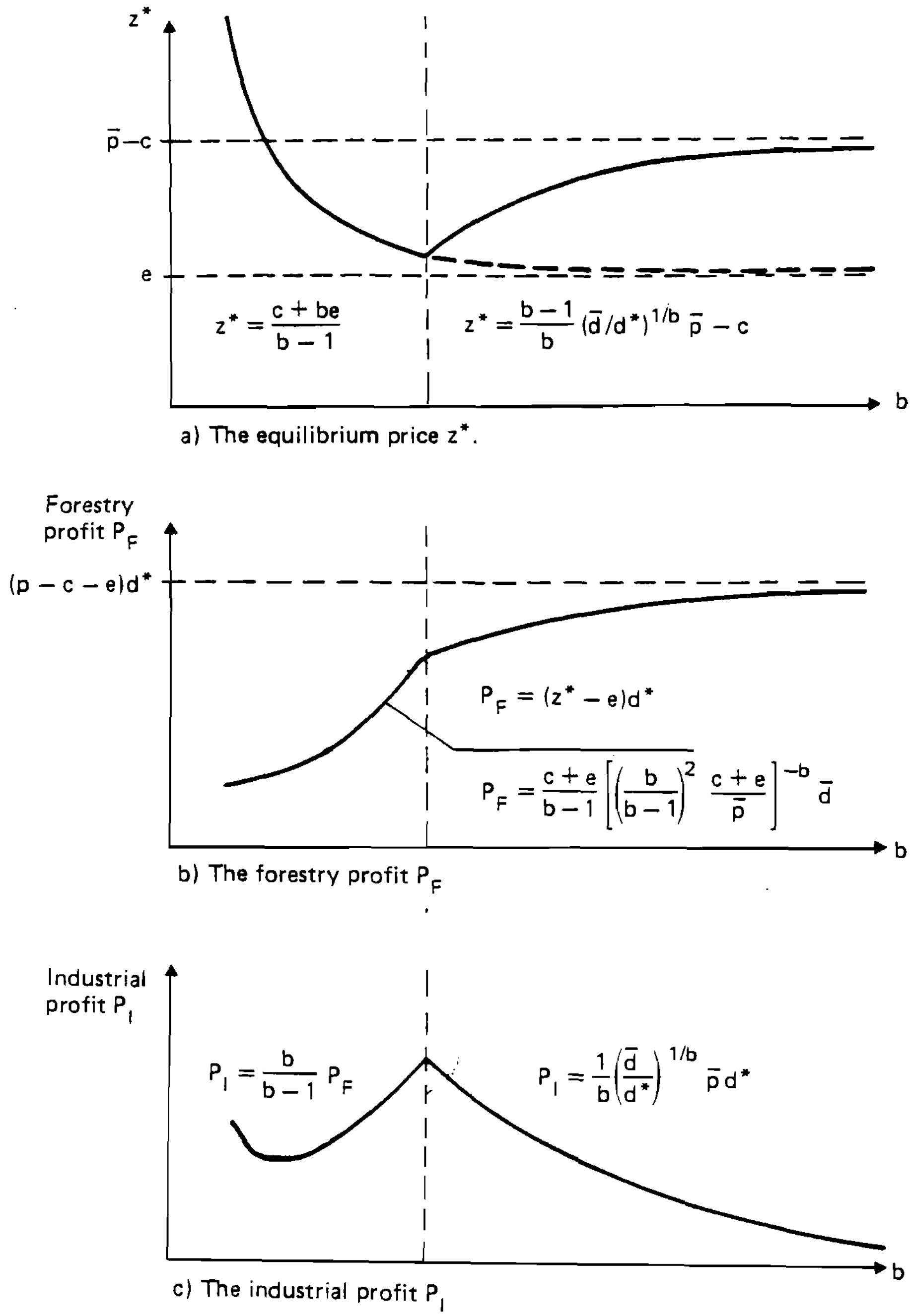
As for forestry profit, (6.3) gives us

$$(7.9) \qquad P_F = (z^* - e)\, d^*$$

which asymptotically approaches $(\bar{p} - c - e)d^*$ (the maximum profit of the entire forest sector) as b approaches infinity.

In Figure 5a we present the equilibrium price z^* of raw wood as a function of b. Figures 5b and 5c show the behavior of forestry profit P_F and industrial profit P_I as functions of b, respectively.

Figure 5. Equilibrium prices and profits as function of the price elasticity coefficient b for the single product - single timber assortment case.



7.2 The Seven Products - Two Timber Assortments Case

For each wood price vector z, the profit maximizing solution for industry, and thereby wood demand y(z), can be expressed analytically. Thus the problem of determining the equilibrium price z^* can be stated as an explicit nonlinear programming problem (6.9) - (6.10) with nonlinearities both in the objective and in the constraints.

We shall redefine the variables so that the resulting problem has nonlinearities only in the objective. Let the inverse function of y(z) be defined as

$$(7.10) \qquad z = g(y) \quad .$$

Substituting this into (6.9) - (6.10) yields the following problem with linear constraints

$$(7.11) \qquad \max_y \bar{P}_F(y) = (g(y) - e)y$$

subject to

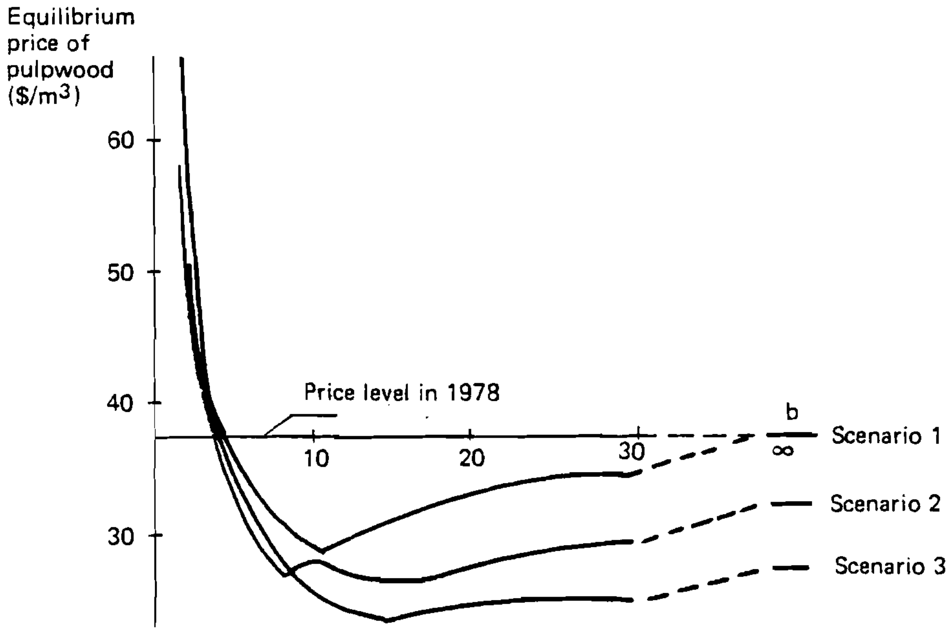
$$(7.12) \qquad R\, y \leq s \quad .$$

For moderate values of b we can solve this problem using standard nonlinear programming codes. The MINOS code /3/ was employed in this study.

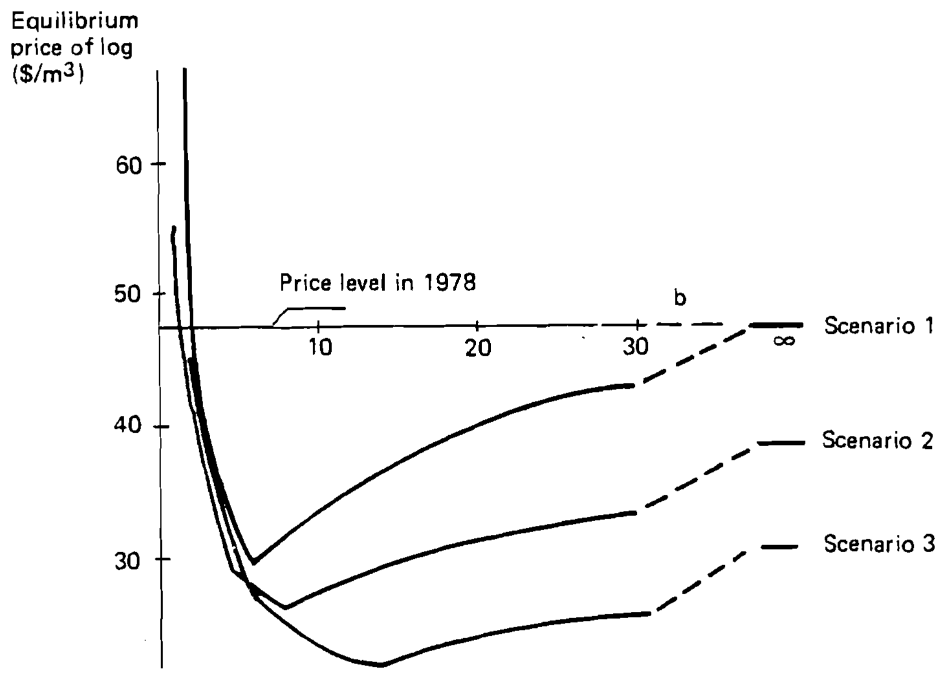
Since we only know g(y) through its inverse function, the following procedure was implemented for evaluating the gradient: (i) Employing iterative methods, solve for the price vector z corresponding to the current value for y; (ii) determine the Jacobian matrix $E(z) = (\partial y_i(z) \,/\, \partial z_j)$ for current y and z, and finally, (iii) calculate the gradient as $\nabla_y P_F(y) = \nabla_z P_F(z)\, E^{-1}(z)$.

For large values of b, the problem is illbehaved and thereby nonsolvable. However, for b = ∞ we obtain the equilibrium price z^* from the dual solution of the following linear program maximizing joint profit for industry and forestry as follows:

Figure 6. Equilibrium round wood prices as functions of the price elasticity coefficient b for world market price Scenarios 1-3.



a) The equilibrium price for pulpwood



b) The equilibrium price for log

(7.13) $\max_{d,y} \; (p - c - eU)d$.

subject to

(7.14) $Ud - y = 0$,

(7.15) $R\,y \leq s$,

(7.16) $d,\, y \geq 0$.

Proposition: Assume problem (7.13-16) to be nondegenerate. If μ^* is the dual optimal solution corresponding to constraint (7.14), then $z^* = e+\mu^*$ is the Stackelberg equilibrium price for $b = \infty$.

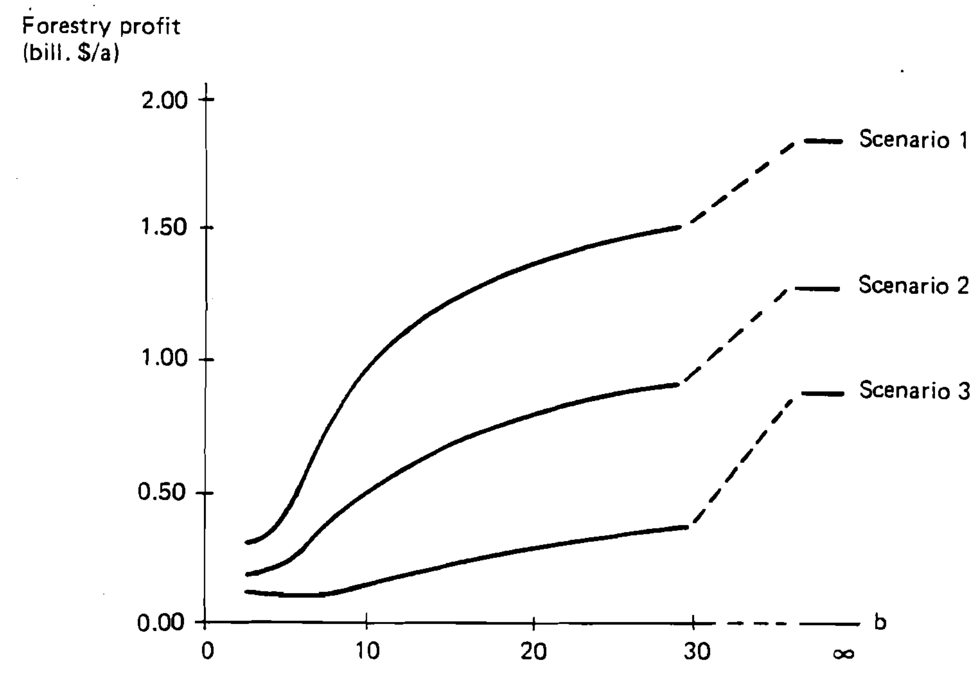
When forestry sets the stumpage price at μ^* and $y \leq y^*$ (the optimal wood consumption) it will maximize its earnings, which, in this case, are equal to the total profit for the entire forest sector. - For a proof of the Proposition, see Appendix 2.

In Figures 6a and 6b we have the equilibrium wood prices as functions of the elasticity parameter b for the three world market price scenarios.

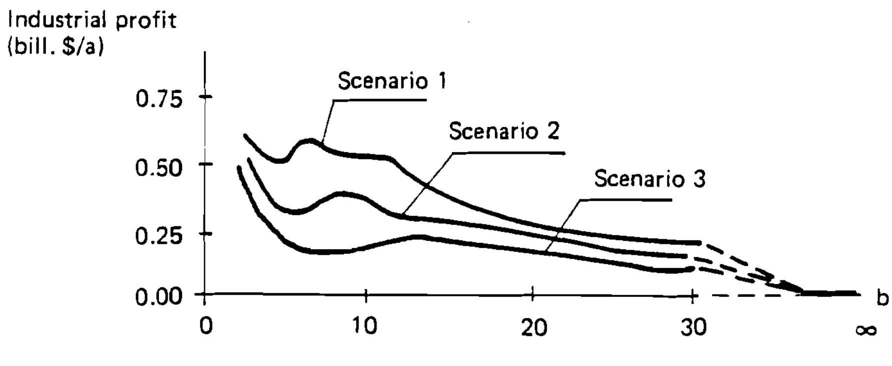
Figures 7a and 7b show the profits for forestry and for industry at the equilibrium. For large values of b (ie $b = \infty$), forestry, absorbs the total profit of the sector. (Note that the necessary return on capital has been taken into account as a cost factor for the forest industry. Zero profit for industry means, therefore, that return on capital equals this minimum.)

For b = 10, 20, 30 and ∞ , the numerical results have been given in Table 3.

Figure 7. Equilibrium profit for the forestry and the industry as a function of the price elasticity coefficient b for world market price Scenarios 1-3



a) Forestry profit



b) Industrial profit

Table 3. Equilibrium prices for pulpwood and log as compare d to current prices when b equals 10, 20, 30 and ∞ .

b = 10		Current Price ($/m^3)	Equilibrium Price ($/m^3)		
			Sce 1	Sce 2	Sce3
Wood Price	Log	48	34	28	23
	Pulpwood	38	28	28	26
Stumpage Price	Log	30	17	11	5
	Pulpwood	15	6	6	3

Table 3a. The case of b = 10.

b = 20		Current Price ($/m^3)	Equilibrium Price ($/m^3)		
			Sce 1	Sce 2	Sce3
Wood Price	Log	48	41	32	24
	Pulpwood	38	33	28	25
Stumpage Price	Log	30	23	14	7
	Pulpwood	15	11	5	2

Table 3b. The case of b = 20.

b = 30		Current Price ($/m^3)	Equilibrium Price ($/m^3)		
			Sce 1	Sce 2	Sce3
Wood Price	Log	48	43	34	25
	Pulpwood	38	34	29	24
Stumpage Price	Log	30	26	16	8
	Pulpwood	15	12	7	2

Table 3c. The case of b = 30.

b = ∞		Current Price ($/m^3)	Equilibrium Price ($/m^3)		
			Sce 1	Sce 2	Sce3
Wood	Log	48	48	39	31
Price	Pulpwood	38	38	33	28
Stumpage	Log	30	30	21	14
Price	Pulpwood	15	15	10	5

Table 3d. The case of b = ∞ .

Generally, we conclude that the current price levels for pulpwood and log are much higher than the equilibrium prices resulting from our analysis. On the other hand, there are substantial differences between prices resulting from the different price scenarios.

8 Summary and Conclusions

In the first part of this paper a (discrete time) dynamic linear model for the forest sector was discussed. The steady state version of it was analyzed in more detail. An application of the steady state forestry model was carried out for the Finnish forests. As a result, alternative sustained yield solutions for the Finnish forestry were obtained.

In the second part of the paper, a steady state sectorial model was adopted to carry out a Stackelberg equilibrium analysis for the round wood market of Finland. Further elaboration was needed for the steady state model until it became suitable for this game theoretic analysis. This elaboration involved definitions of objective functions for the forestry and for the industry.

For the industrial model, a demand function with a constant price elasticity coefficient b was chosen for each product. A reasonable assumption conserning the value of b is in the range between 10 and 30. If b is greater than 30 we price ourselves out of the market with a 10 percent increase in price. On the other hand, when b is under 10 the demand is very rigid; in other words, changes in price do not affect demand, which does not correspond to the present market situation. However, sensitivity analysis was carried out on the whole range of $1<b\leq\infty$. The other highly uncertain and sensitive figure in the analysis is the world market price (defined as sales price when b approaches infinity). For sensitivity analysis, three price scenarios were constructed for each forest product as follows: (1) An optimistic world market price is defined as total production cost in Finland (including wood cost at present prices and a ten percent real interest on total invested capital), (3) a pessimistic world market price is defined as being roughly equal to the production cost of our major future competitors in the world market, and (2) a likely scenario which is the average of the two above.

As the numerical results presented in Section 7 show the current price levels for pulpwood and log are much higher than the equilibrium prices resulting from our analysis. On the other hand, there are substantial differences between prices resulting from the three price scenarios for the world market prices of wood products.

APPENDIX 1.

Proposition: For an optimal solution of the forestry problem (F) defined on page 6 there is an age group A such that $h(a) = 0$, for all $a \neq A$, and $w(a) = 0$, for all $a > A$.

Proof: Clearly, for an optimal solution $w(1) = p > 0$. Let $a = A$ be the smallest age group for which $w(A+1) = 0$. Then $h(A) > 0$ and $w(a) = h(a) = 0$ for all $a > A$. To show that $h(a) = 0$ for $a < A$, we consider the optimal basis for (F) partitioned as follows:

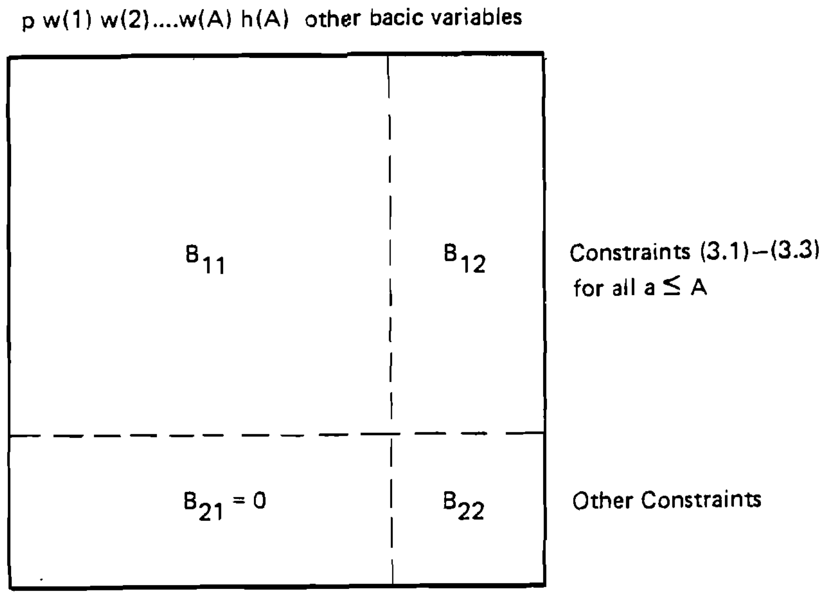


Figure: An optimal basis matrix for (F).

Here B_{11} is square and $B_{21} = 0$. Thus, B_{22} is nonsingular and therefore, $h(a)$ is nonbasic for $a < A$.||

APPENDIX 2.

Proposition: Assume problem (7.13-16) to be nondegenerate. If μ^* is the dual optimal solution corresponding to constraint (7.14), then $z^* = e+\mu^*$ is the Stackelberg equilibrium price for $b = \infty$.

Proof: Consider the problem (G) of maximizing the profit for the entire forest sector:

(G.1) $$\max_{(d,\, y) \geq 0} (p - c - eU)d$$

subject to

(G.2) $$Ud - y = 0$$

(G.3) $$Ry \leq s$$

Let (d^*, y^*) be the optimal primal solution for (G), and (μ^*, λ^*) the optimal dual multipliers (for constraints (G.2) and (G.3), respectively). Let $\varepsilon > 0$ and define a wood price vector $z(\varepsilon) = e + (1-\varepsilon)\mu^*$. For this wood price the profit maximization problem (I) of industry is the following:

(I.1) $$\max_{(d,\, y) \geq 0} (p - c - z(\varepsilon)U)d$$

subject to

(I.2) $$Ud - y = 0$$

(I.3) $$Ry \leq s$$

Optimal primal and dual solutions for (I) are denoted by (d', y') and (μ', λ'), respectively.

To prove the proposition, we shall show that an optimal solution (d', y') for (I) is optimal for (G) as well, and that the profit thereby obtained by forestry can be made arbitrarily close to the optimal profit for (G), the profit for the entire sector. The latter is achieved when ε approaches zero corresponding to the limiting wood price $z(0) = e + \mu^*$.

One can readily check the optimality conditions for (I) and observe that the primal and dual solutions (d^*, y^*) and $(\varepsilon\mu^*, \varepsilon\lambda^*)$, respectively, are optimal for (I). Because of the primal nondegeneracy assumption for (G), and thereby for (I), the dual optimal solution for (I) is unique. Therefore $(\mu', \lambda') = (\varepsilon\mu^*, \varepsilon\lambda^*)$. This together with the optimality condition for (I) applied to the primal solution (d', y') and the dual solution (μ', λ'), imply the optimality conditions of (G) for (d', y') and (μ^*, λ^*), ie an optimal solution (d', y') for (I) is optimal for (G) as well. From the optimal profit $(p - c - eU)d^*$ of (G), an amount of $\varepsilon\lambda^* y^*$ belongs to the industry, and this share approaches zero with ε . ||

REFERENCES

/1/ **JAAKKO POYRY INTERNATIONAL OY** (1979) Improving the International Competitiveness of the Finnish Forest Industry (in Finnish). Helsinki, Finland.

/2/ **KALLIO, M., A. PROPOI AND R. SEPPALA** (1980) A Model for the Forest Sector. WP-80-34. Laxenburg, Austria: International Institute for Applied Systems Analysis.

/3/ **MURTAGH, B. AND M. SAUNDERS** (1978) Large-Scale Linearly Constrained Optimization. Mathematical Programming 14 41-72.

/4/ **STACKELBERG, H. VON** (1952) The Theory of the Market Economy. Oxford University Press, Oxford, England.