Stochastic Hydrology: An Introduction to Wet Statistics for Dry Statisticians

Lloyd, E.H.

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This paper is concerned with some aspects of stochastic modeling in hydrology which are of fundamental importance for planning, design, and operation of water resources systems. The Author points out that in spite of the rich growth of stochastic modeling in this field that has occurred in the last decade, the emphasis is still more on the probabilistic model than on the "statistical questions to which the model gives rise". The additional research needs are stressed.

Janusz Kindler
Regional Water Management
Task Leader

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STOCHASTIC HYDROLOGY: AN INTRODUCTION TO WET STATISTICS FOR DRY STATISTICIANS

Emlyn Lloyd

1. WHAT HYDROLOGY IS ABOUT

Hydrology is, evidently, to do with water. Amplification of that trite remark reveals a science of daunting scope. According to one of the standard definitions [Ad Hoc Panel on Hydrology (1962)], "Hydrology is the science that treats the waters of the Earth, their occurrence, circulation, and distribution, their chemical and physical properties, and their reaction with their environment, ..." Another definition [Winsler and Brator (1959)] sets out the subject as "... the science that deals with the processes governing the depletion and replenishment of water resources of the land areas of the earth. It is concerned with the transportation of water through the air, over the ground surface, and through the strata of the earth."

It is a subject with a respectable history, dating back at least 3700 years to the irrigation problems of Hammurabi's Babylon [Neumann (1980)]. While certain facets of hydrology are still mainly descriptive, others have developed a considerable technology. The (still large) empirical content is being fertilized by the overlaps that have come into being with physics, chemistry and hydraulics. During the past three decades the whole area has increasingly been subjected to theoretical and mathematical treatments of one sort or another.

Hydrology has enormous and obviously practical importance to mankind, in the supply of water for domestic, industrial and agricultural uses, as well as for power generation, for the alleviation of the effects of floods and droughts, for river navigation, etc. It is a very large employer of technically qualified manpower, particularly of those with a civil engineering background, and is related to huge capital outlays for the construction of dams, reservoirs, weirs, aquaducts, levees, and so on.

All advanced countries have scientific and professional hydrological organizations, and there are many international organizations as well. Of these, the one most relevant to this article is the International Association of Scientific Hydrology (IASH). The principal periodicals which publish (among other papers) work of a stochastic nature are the IASH Bulletin, the Journal of Hydrology, Water Resources Research, and Advances in Hydrology.

To obtain a rapid view of what professional hydrologists currently regard as standard equipment, an examination is recommended of the chapter headings in the "Handbook of Applied Hydrology" [Ven Te Chow (1964)]. As well as sections on Meteorology, Fluid Mechanics, Runoff, Droughts, Quality of Water, Hydrology of Flow Control, Water Law, Water Policy, etc., there is a chapter on Statistical and Probability Analysis in Hydrologic Data. Significantly, this is the longest chapter in the Handbook.

2. THE STOCHASTIC NATURE OF HYDROLOGY

It is, no doubt, to the stochastic nature of rainfall that the weather owes its popularity as a subject for conversation, and this high degree of apparently random variability is shared by all aspects of the hydrologist's art.

This was obvious to most hydrologists from the start - not, admittedly to quite all: there were some who were reluctant to admit the idea of chance events - but the complexity of the phenomena militated against the early adoption of statistical methods into professional practice. In addition (and this is the feature that most profoundly impresses itself on the newcomer to hydrology) there is a widespread paucity of data. All but the most trivial problems are concerned with multivariate spatio-temporal variables, and the available time-series data usually consist of short runs only: a 40-year record of river flow is a good deal more than one can usually hope for, and 40 years is a very short time in the geophysical world. Not only this, but the data themselves are often unreliable. Until recently the only flow information related to the "stage" of the river at the observation point, i.e. the height of the river surface above a given reference level. To convert stage into flow rates, one needs a calibration that depends on the cross-section of the channel itself a fluctuating and ill-recorded entity.

(When the present author first lectured on statistical methods to an audience of hydrologists he referred to the telemetering network what he naively assumed to exist, automatically flashing a continuous record of flow rates to a central recorder. He was greeted with coarse laughter.)

When one adds to the above the information that it is often the worst-documented aspects of their inadequate data runs that chiefly concern hydrologists (typically the tail of a distribution) the total picture that emerges is somewhat sombre.

To give a simple example, the annual flood season flows of a river at a given location have a positively skewed distribution whose c.d.f. $F(x)$ is known only to a somewhat rough approximation. A flood flow of specified magnitude x_o (somewhere in the upper tail) will be exceeded with probability $p = 1 - F(x_o)$. Hydrologists speak in terms of the "n-year flood." That is the flow x_o whose recurrence time T_n has expectation $1/n$. What hope has one of estimating the 100-year flood from a 40-year record? Less hopelessly, what is the distribution of T_n? The latter is an important question since, if civil engineering works are to be installed, designed to withstand, say, the 20-year flood, the chance of their being overtopped in a given year is a matter of high socio-economic (and possibly legal) interest. The computation will involve assumptions about the form of F, particularly its upper tail, as well as the autocorrelation structure and the local stationarity or otherwise of the time series of annual flows. Pure statistical questions become confounded with matters of judgement and guesswork to which the model may well be quite sensitive.

That hydrologists' preoccupation with stream-flow distributions is long-standing and continuing, may be verified by examining some of the standard books [e.g. Fiering (1967)] and the contents of recent issues of the standard periodicals such as the current volume of Water Resources Research [e.g. Todini (1980)].

To go back a few years, it might be mentioned that the American hydrologist, Hazen, was one of the first (if not the first) to advocate the use of probability graph paper: "The practical difficulty of plotting ... is the great curvature of the lines showing the required storage. This difficulty... has been removed by using paper ruled with lines spaced in accordance with... the normal law of error" [Hazen (1914)]. Another statistical first or near-first was Sudler's early advocacy of the use of statistical simulation [Sudler (1927)]: "Using an appropriate curve ... the theoretical run-off of all the years of a stated period may be ascertained and by chance selection of these values, an artificial record may be constructed." Adoption of a stochastic approach was not to be taken for granted for several decades. When it came, it was (not surprisingly) in the form of the straightforward adaptation to hydrological purposes of the standard canon of statistical techniques of the time. The encyclopaedic "Handbook of Applied Hydrology" referred to earlier [Ven Te Chow (1964)], for example, presents an implied description of the accepted state-of-the-art in the early sixties, with its catalogue of standard probability distributions and their lower moments, its account of estimation and curve fitting, of regression and correlation analysis, analysis of variance and of covariance, and - of prime importance then and since to the practising professional - time series analysis. This is carried out in terms of trends (taken care of by moving averages), seasonal and other periodicities (harmonic analysis), and tests for the significance of estimated correlations. The chapter continues with a brief account of moving-average and autoregressive sequences, and ends with an introduction to the simulation techniques that were subsequently to play such a large part in hydrologic research.

It might be added that the statistical expertise displayed had been achieved by a boot-strap operation on the part of hydrologists and engineers, statisticians outside the profession having - with a few honourable exceptions - shown a curious reluctance to interest themselves in these applications.

At about the same time as Ven Te Chow's Handbook there appeared the Harvard school's influential "Design of Water Resource Systems [Maass, et al., (1962)]. This was designed not as an encyclopaedia of accepted practice but as a deliberate attempt to inform the hydrological world of newly developed concepts and methods. The emphasis was on a "Systems" approach, integrating technology with economic and social cost-benefit analyses. The techniques propounded were mainly deterministic but were often presented in the (then relatively novel) form of algorithms, flow-charts and computer programs. Mathematical models involving deterministic programming played a large part. This emphasis on optimization and systems analysis has led to the situation where contemporary periodicals frequently carry mathematically sophisticated papers involving, for example, Kalman filter techniques [e.g. Bolzern, Ferrario and Fronza (1980)].

In addition to that kind of applied mathematics, however, the Maass opus also contain a chapter on the "synthesis" (hydrologese for simulation) of stream-flow sequences for the analysis of river basins by computer experiments that marks a forward significant step of a stochastic nature. Realizations of seasonally varying flow distributions with seasonal auto-correlations were dealt with by means of seasonal autoregressive schemes, fed by random numbers taken from published tables of random digits (it being still too early for computer-generated pseudo-random numbers).

A more consistently stochastic viewpoint in hydrology was presented by an equally influential book on the application of stochastic processes by the Russian writer, Kartvelishvili, an English translation of which appeared in 1969 [Kartvelishvili (1969)]. This was followed by Yevjevich's book on stochastic processes [Yevjevich (1972)] based mainly on American experience and expressing American research activities. A few years later a translation into English of Kaczmarek's (Polish) book on statistical methods appeared [Kaczmarek (1977)].

The more recent work on stochastic hydrology is available only in the form of disseminated research papers or in the proceedings of conferences (to which hydrologists, fortunately for us, are rather addicted).

3. STOCHASTIC MODELING IN HYDROLOGY

It would be impossible to do justice to recent theoretical work on the space available here, but there is one outstanding characteristic which must be remarked on. This is the astonishingly rich growth of stochastic modeling that has occurred, the emphasis being more on the probabilistic model than on the statistical questions to which the model gives rise. This is not to say that there has not been a continuation of traditional engineering methods and bread-and-butter statistics, with much use of regression techniques, and a development of applications of more sophisticated classical multivariate statistics such as principal component analysis [see, e.g. Morin et al., (1979)]. This healthy development proceeds, but the new quality of the past few decades has been the modeling mentioned above.

To quote the anonymous author of the Introduction to a recent volume of Conference Proceedings [Coriani, Maroni and Wallis (1977)] "... the building of models has outpaced their use in specific water resource planning and management activities. The lag in the use of specific models may be attributed to many factors, among them being that planners and managers are unaware of recent developments in mathematical models, reluctant to use more sophisticated models when simpler ones seemingly suffice, or lack of understanding and competence in the use of advanced models. On the other hand, model builders have not always understood the problems faced by planners and managers, and have not in general constructed their models in ways that facilitate decision-making."

3.1 The Hurst Effect

The flavour of the kind of research referred to may perhaps be conveyed by outlining the recent history of long-term stream-flow investigations.

A good stochastic model for the sequence of flows at a given location is a matter of prime importance to hydrologists. It is on this model [see, for example, Fiering (1967), and Todini (1980)] that he has to base his calculations for reservoir design, flood prevention, and so on. Of particular technical interest are not only the seasonal and annual means, variances and skewnesses of the flows, but also the magnitudes and fluctuation patterns of annual maxima and minima. The failure of consecutive monthly flows to display mutual independence, and even more importantly, the tendency of wet years (and high flows) to occur in groups, and for dry years (and low flows) to occur in groups, is called persistence, and every clue to the pattern of persistence and its effects must be utilized to the full in creating the model on which the (unavoidable) simulation studies are to be based.

Such a clue, which caused the greatest excitement, was announced to the world in 1951 by the British engineer, H.E. Hurst, and elaborated by him in subsequent years [Hurst (1951), (1956), (1957) and (1965)]. Hurst had spent his professional life in Egypt, in charge of studies of the Nile. It was from this work that the "Hurst effect" was discovered, but Hurst went on to show it to be a feature of most rivers (and, indeed, of many other geophysical time series).

To understand what the Hurst effect is, imagine an arbitrarily large reservoir of rectangular section, whose initial contents define a conventional zero level. Annual inflows $x_1, x_2, \ldots, x_n$, in the absence of any withdrawal, would bring the water content after n years to a level $s_n = \sum_1^n x_r = n\bar{x}_n$. A constant annual withdrawal rate over the n years, of magnitude $\bar{x}_n$, would leave the system at the same level at which it started. (This simple but powerful concept is due to an Austrian engineer appropriately named Rippl [Rippl (1883)].) In the j-th year, $j = 1,2,\ldots,n$, the quantity ${}_nS_j^* = s_j - j\bar{x}_n$ (where $s_j = \sum_1^j x_r$) represents the height of the water level above (if positive) or below (if negative) the conventional zero mark. Consequently the magnitude of

$$u_n^* = \max_{j=1,2,\ldots,n} ({}_nS_j^*) \quad ,$$

represents the lowest height of the reservoir walls consistent with there being no overflow during the n-year period, and

$$-\ell_n^* = - \min_{j=1,2,\ldots,n} ({}_nS_j^*) \quad ,$$

the least required depth below the conventional origin consistent with the maintenance of the desired annual withdrawal rate. The smallest reservoir volume consistent with no overflow and no failure of yield during the n years in

$$r_n^* = u_n^* - \ell_n^* \quad ,$$

the so-called adjusted Hurst range. This may be expressed in non-dimensional form by dividing by the sample standard deviation d_n of the inflows, leading finally to the rescaled adjusted Hurst range

$$r_n^{**} = r_n^*/d_n \quad .$$

Although this quantity arises in a natural way in connection with reservoir design, the sampling distribution of r_n^{**}, as a function of the time duration n, is of more general interest. Denoting by R_n^{**} the random variable of which r_n^{**} is a realization, Hurst's empirical law may be interpreted as stating that

$$E(R_n^{**}) \propto n^h \;, \qquad n < \text{ca, } 1000,$$

where the "Hurst exponent" h is a constant whose value is about 0.73. Elementary models (see below) would suggest a law of the form $n^{1/2}$, and it is the fact that $h \neq 1/2$ that constitutes the Hurst effect. (The possibly mysterious 1000 in the above formula refers to the fact that Hurst's longest data run, giving the history of the Nile is about 1000 years long.)

The Hurst exponent was taken to be a significant characteristic of geophysical time series in general, and hydrologists soon expressed a desire to build this into their simulation models. Mathematicians were attracted. Difficulties consequent on the apparent mathematical intractability of the distribution of R_n^{**} were initially disposed of by the familiar mathematical device of attacking a simpler related problem. Thus, in 1951, Feller deemed expressions for the asymptotic expectation of the "crude Hurst range" R_n, defined as

$$R_n = (\max_j - \min_j)\ (\sum_1^j X_n) \quad ,$$

and of the "unrescaled" adjusted range R_n^*, after replacing the input sequence $\{X_\varepsilon\}$ by an approximating diffusion process. Not surprisingly he showed [Feller (1951)], for both the crude and the adjusted range, that the asymptotic expected value was proportional to $n^{1/2}$. The exact value of the expected crude range $E(R_n)$ of a sequence of i.i.d. Normals was next. [Anis and Lloyd (1953)] discovered to be proportional to $\sum_1^n r^{-1/2}$, a quantity which is asymptotic to $n^{1/2}$, and this work was later extended, to the adjusted range $E(R_n^*)$, [Solari and Anis (1957)], again of course asymptotically $n^{1/2}$, the corresponding result for the rescaled adjusted range of i.i.d. Normal increments, the result required for direct comparison with Hurst's empirical law, defied attack until 1976, when it was shown [Anis and Lloyd (1976)] that

$$E(R_n^{**}) = \frac{\Gamma\{1/2(n-1)\}}{\sqrt{\pi}\ \Gamma(1/2n)} \sum_{r=1}^{n-1} \left(\frac{n-r}{r}\right)^{1/2} \quad .$$

This shows Hurst-like behaviour for quite small values of n only, the local Hurst exponent d log $E(R_n^{**})$/d log n decreasing fairly rapidly from 0.65 at n = 5 to 0.54 at n = 100, falling off thereafter to the asymptotic value of 0.5.

This work put paid to any lingering ideas that independent Normal summands would display Hurst-like behaviour. Could the explanation of the Hurst effect lie in the shape of the distribution of the summands? In principle, yes. Moran (1964) pointed out that, for the crude range at least, Hurst-like behaviour would be displayed by independent summands having a stable distribution. (This property depends on the result that if, for example, the summands X_r are independently distributed in the symmetrical stable form with index γ, the sum $S_k = \sum_1^k X_r$ has, for $k = 1,2,\ldots$, the same distribution as $k^\alpha S_i$, with $\alpha = 1/\gamma$, so that the distribution of $k^{-d} S_k$ is independent of k.) The extension of this Hurst-like behaviour to the adjusted range was established by Boes and Salas-La Cruz (1973).

Interesting though this was, it did not persuade hydrologists that they ought to model their flows in terms of stable variables. Moran (1964) had pointed out that gamma variables of sufficiently high skewness would have to some degree the heavy-tail property to which the Hurst-like behaviour of the stables seemed intuitively to be attributable and could be expected to exhibit the Hurst-effect over an acceptably long pre-asymptotic interval, but the degree of skewness required turned out to be unrealistically high [Anis and Lloyd (1975)].

So one turns to the autocorrelation structure. At this point an element of fantasy enters the story. Mandelbrot (1965) introduced a brilliantly constructed Gaussian process that exhibited the Hurst effect, the so-called "fractional Gaussian noise." This is a continuous-time process S(t) which has the "self-similar" property that, for specified α, $0 < \alpha \leq 1/2$, the process $t^{-\alpha}S(\alpha t)$ is time-independent. (Compare the Stables.) In this sense the Brownian movement $B(t)$ is self-similar with parameter $\alpha = 1/2$: for $B(\alpha t)$ is Normal $(0,\alpha t)$, whence $(\alpha t)^{-1/2}B(\alpha t)$ is Normal (0,1), and so independent of t, with exponent $\alpha = 1/2$. Mandelbrot's process has the advantage of possessing this property for arbitrary α, $0 < \alpha \leq = 1/2$, for arbitrarily large values of t. Explicitly this process, $B_k(t)$, say, is defined as an additive one with independent increments given by

$$B_k(t_2) - B_k(t1) = a_k[S_{-\infty}^{t_1}\{(t_2-s)^{k-1/2}-(t_1-s)^{h-1/2}\}dB/s)$$
$$+ S_{t_1}^{t_2}(t_2-s)^{h-1/2}dB(s)] \quad , \quad t_2 > t_1 \quad .$$

This may be regarded as having been derived by the usual limiting process from a discrete moving average scheme. It is a mathematical entity of the highest interest, but, as was remarked by the present author [Lloyd (1974)] "as an algorithm for computing realizations of Hurst-like sequences this process has the serious disadvantage (a consequence of its slow convergence) of requiring extremely large computer capacity.... There are those in the hydrological world who profess to find difficulties in visualizing a physical process that could in fact plausibly be described as fractional Gaussian, and from this fact a certain amount of controversy has arisen." Mandelbrot and Wallis (1969) ascribe the Hurst-like behaviour of the process to its "long memory", and the element of fantasy referred to earlier lies (in the writer's opinion) in the belief that remotely distant realizations of a geophysical time series could perceptably influence its present fluctuations.

Truly amazing quantities of computer time have been devoted to this process and approximations thereto [e.g. Mandelbrot and Wallis (1969)], its relation to realistic hydrology becoming a little diffuse. It is presumably this kind of dehydrated hydrology that has induced respected pioneers of stochastic hydrology to voice their disquiet at what they regard as numerology [Nash (1978)] and at what they see as the excessive zeal of desk-top theoreticians. Thus Fiering (1976) says: "Fascination with automatic computation has encouraged a new set of mathematical formalisms simply because they can now be computed; ... flow synthesis and systems simulation have become common methodologies in water-resource design. Neither is used to generalize results but rather to make highly specific estimates of system performance when alternative systems are defined and tested. ... But it is well-known that data is not available at precisely the locations where the simulation needs to test system performance. Thus it must be transferred ... from gauged to ungauged locations...." He goes on to point out that the model error associated with the transfer of information to ungauged sites is often so great as to cast severe doubt on the validity of the results, which tend nevertheless to be uncritically calculated to several significant figures.

Realistic engineers have since developed simple ARMA models for generating sequences having Hurst-like behaviour [O'Connell (1971)].

The likely physical explanation of the Hurst effect is now however thought to lie in the unavoidable heterogeneity imposed on historical data by occasional shifts of origin (as must certainly have occurred during the 1000-years of the Nilometer record) or displacements of the measuring equipment (as in occasional redesigning of rain-gauge networks). Such an explanation is consistent with Hurst's own suggested model [Hurst (1957)], which visualizes a process in which the summands are independently distributed about a mean which is itself subjected to jumps of random magnitudes at random times. This has been shown by extended computer experiments by Klemes (1974) and others to have the required properties. [For a recent review, see Boes and Salas (1978).] Analysis of such meteorological time series showing pronounced Hurst-like behaviour has recently shown that the relevant data were indeed contaminated in this way [Potter (1970)]. In a similar view of scepticism, Klemes and Bulu (1979) have poured some refreshingly cold hydrological fluid on the wilder flights of stochastic fancy.

3.2 Short-Term Run-Off Models

Of equal importance with the long-term pattern of annual, seasonal or monthly flows is the problem of predicting the immediate and short-term run-off generated by a single rainstorm in a given catchment, and the associated problem of "routing" the ensuing water through the channels and other storage and alternating devices available. The time lags differentially imposed on the rain-induced run-off by the nature of the surface and subsurface soil and its percolation properties invite the use of geophysically based models in conjunction with the resources of classical hydraulics. Given that the rainfall incidence is stochastic, the working techniques in standard use tend to be largely empirical models of a partly deterministic and partly probabilistic nature. One interesting model [Nash (1957), (1958), (1960)] treats the catchment as a sequence ("cascade") of linear reservoirs, the outflow from each constituting the inflow to the next. (A "linear reservoir" is a conceptual storage system, from which the outflow is a linear function of the quantity of water contained in it.)

Stochastic linear reservoirs have been studied by Moran (1967) and by Anis, Lloyd and Saleem (1979) and others.

Stochastic models of rainfall generation [Amorocho and Lloyd (1978)] and run-off [Weiss (1973)] are now beginning to appear. In the Amorocho model, rain is produced by precipitation cells which come into existence at random epochs on a spatially three-dimensional Poisson process (itself moving at uniform speed relative to the ground), each cell's rain-producing capacity growing spatially and in intensity and then dying away. In its simplest form the Weiss run-off model is an application of a filtered Poisson process to the problem of simulating the hydrograph (viz. the run-off as a function of time) generated by a brief concentrated rainstorm. The hydrograph typically looks like a positively skewed probability density function superimposed on a more or less constant "base flow," the skew shape resulting from a rapid initial increase in run-off followed by a slower recession. A continued record of real flows will show a succession of such shapes, with maxima of varying heights occurring at apparently random epochs. Weiss's "shot-noise" model gives the flow at time t as the sum of pulses of the form $\delta(t,T_m)\ Y_m\text{-exp-}b(t-T_m)$, the jumps Y_m being i.i.d. exponential variables and their epochs of occurrence T_m being generated as a Poisson process. (Here the δ function equals unity if $t > T_m$, and equals zero otherwise.) Elaboration of the model involve the superposition of several independent shot-noise processes having differing Poisson rates and recession constants. [For a brief review, see O'Connell (1977)].

3.3 Stochastic Reservoir Theory

The third and final example of stochastic modeling to be touched upon in this brief survey concerns water storage. Stochastic reservoir theory owes its existence to Moran [(1954), (1955), (1959)]. The model is basically one in which a sequence of random variables is fed into a finite reservoir from which water is released in accordance with a "withdrawal policy" which may depend on current and past inflows and storage values. In the Moran reservoir these inflows are i.i.d. or independent but seasonally distributed. For i.i.d. inflows and constant withdrawals the sequence of storage values is a lag-1 Markov Chain.

The model turned out to be the ancestor of a branching process with markedly non-identical offspring. One such descendant was the influential R.S.S. symposium on storage systems in which Gani (1957) summarized the findings of Moran's school and D.G. Kendall (1957) introduced to a delighted mathematical world the charms of <u>dam theory</u> (effectively infinite reservoirs with inflows consisting of continuous-time processes with independent increments, with unit withdrawal policy), in a paper which was itself the highly fecund parent of a large progeny of research activity. Other evolutionary lines have concerned themselves less with mathematical elegance and more with attempts at engineering realism, in particular the adaptation of Moran's original model to the case of autocorrelated inflows. Lloyd (1963) for example produced a modification allowing Markovian inflows, in which the joint distribution of inflows and storage values is bivariate first-order Markov. [See also Langbein (1958), Phatarfod and Mardia (1973), Phatarfod (1976).]

At the time of writing, however, no satisfactory stochastic model has appeared capable of dealing with interconnected systems of multipurpose (flood-prevention/over-year storage/ seasonal storage/power generation) reservoir; much work remains to be done.

4. APOLOGIES

The foregoing is a brief, partial and incomplete picture of some aspects of stochastic hydrology, written for statisticians who have not yet got their feet into the water. It was written in the hope of conveying to them some idea of the attractiveness of stochastic hydrology: so - statisticians and applied probabilists, if you are sighing for fresh worlds to conquer - come on in! Hydrology is waiting for you!

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