International Institute for Applied Systems Analysis
IIASA
www.iiasa.ac.at

The Accuracy of Population Projections

Stoto, M.A.

IIASA Working Paper

WP-79-075

August 1979

THE ACCURACY OF POPULATION PROJECTIONS

Michael A. Stoto

August 1979
WP-79-75

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

ACKNOWLEDGEMENTS

The original calculations for this paper were performed while the author was supported by N.S.F. Grant SOC 75-15702 in the Department of Statistics, Harvard University. A preliminary version of the paper was presented at the annual meeting of the Population Association of America, Philadelphia, Pa., April, 1979.

The author would like to thank his colleagues at Harvard and IIASA for many stimulating discussions on the topic of this paper. Special thanks is due to Nathan Keyfitz, who originally suggested the idea of a historical study of projection accuracy.

ABSTRACT

Population projections are key elements of many planning or policy studies, but are inherently inaccurate. This study of past population projection errors provides a means for constructing confidence intervals for future projections.

We first define a statistic to measure projection errors independently of the size of the population and length of the projection period. A sample of U.S. Census Bureau and U.N. projections indicates that the distribution of the error statistic is relatively stable. Finally, this information is used to construct confidence intervals for the total population of the United States through the year 2000.

THE ACCURACY OF POPULATION PROJECTIONS

Michael A. Stoto

1. INTRODUCTION

Population projections or predictions are basic inputs for both governmental and private planners. The basic question is: How many people (perhaps broken down by age, sex, and so forth) will there be in a certain area at a certain time in the future? Planners can answer this question in many ways, depending on what assumptions they are willing to make. Keyfitz (1972) offers a catalog of the available techniques.

A second question is less frequently asked: Within what range can we be sure the future population will be? This paper answers the second question both theoretically and specifically for the United States in the year 2000.

Keyfitz (1972) points out that even though population projections are simple mathematical extrapolations of current trends and assumptions about the future, they are frequently regarded as predictions. This is especially true for projections issued by Government agencies. Throughout this paper we will regard all projections as predictions, and therefore can talk about the accuracy of population projections.

There are two ways to analyze their accuracy. The first is to specify a mathematical model for the growth of population, and explore the effects of variation in the inputs. The U.S. Bureau of the Census and the Population Bureau of the United Nations do this informally when they present "High", "Low" and "Medium" series of projections, reflecting different beliefs about the future course of mortality and fertility. Sykes (1969), Lee (1974) and Cohen (1976,1977a,1977b) do it more formally by developing mathematical models for the variations in vital rates.

This paper presents a data-analytic approach to the same problem. Rather than making assumptions about either the magnitude of possible error in our assumptions, or a mechanism for the change in rates, we let the projections speak for themselves. In the past two hundred years, competent demographers have made many predictions for target years which have already gone by. A study of the magnitude of their errors will tell us about the possible errors in today's projections.

This article is not intended to criticize or applaud the quality of population predictions. Instead it aims to provide confidence intervals for projections made today, assuming their quality is as good as or better than it has been in the past. We begin by examining the historical record.

2. EXAMINING THE HISTORICAL RECORD

In 1775, on the even of the American Revolution, Edward Wigglesworth (1775) published a pamphlet entitled "Calculations on American Populations". The pamphlet contained, among other things, a forecast that the population of the "British colonies" in 1975 would be 640 million. About 1950, the U.S. Bureau of the Census (1953) made a projection of 210 million for the same date. In 1970, we could have made a very simple projection by assuming that the overall 5 year growth rate for 1970-75 would be the same as it was from 1965 to 1970. The projection would have been 216 million.

We now know that the U.S. population in 1975 numbered 214 million. Therefore we can evaluate each of the three projections. Some results appear in Exhibit 1.

We first calculate the difference between the predicted and actual populations, ΔP. By this criterion, assuming a constant growth rate from 1965-1975 yields the best prediction. This is not surprising; a five year projection should be easier to do well than a 25 or 200 year projection. This indicates one reason why ΔP is not a good measure for projection errors: it does not take the "duration" of a projection into account. Most people would regard the Census Bureau's 1950 forecast with an error of 3.4 million over 25 years as better than the constant growth forecast with an error of 2.5 million in 5 years.

In 1895 Edwin Cannan (1895) forecast the 1951 population of England and Wales as 37.5 million, and it turned out to be 41.2 million. Cannan's error of 3.7 million on an estimate of 37.5 million seems worse than the U.S. Census Bureau's error of 3.4 million on an estimate of 210 million. The second objection to ΔP is that it is sensitive to the population size.

We begin our analysis by defining a statistic, Δr, which takes these two factors -- duration of the projection and total population size -- into account. We then calculate Δr for a number of actual projections to target years which have passed. A statistical study of the distribution of Δr then leads us to statements about the probable size of future projection errors.

3. DEFINITION OF Δr

Constant exponential increase is the simplest model of population growth. According to this theory, if P_0 is the current population, and r is the growth rate, the population T years from now, P_T, is

$$P_T = P_0 e^{rT} . \tag{1}$$

If the growth rate is not a constant, but instead a function of time, $r(t)$, we write

EXHIBIT 1

TARGET: U.S. 1975	PREDICTION	ACTUAL	ΔP	Δr
WIGGLESWORTH (1775)	640,000	213,540	426,460	.55
U.S. CENSUS BUREAU (1950)	210,092	213,540	-3,448	-.07
CONSTANT GROWTH (1970)	216,028	213,540	2,488	.23
TARGET: ENGLAND + WALES 1951				
CANNAN (1895)	37,500	41,200	-3,700	-.17

POPULATION PROJECTION ERRORS

$$P_T = P_0 \exp\left(\int_0^T r(t)\,dt\right) .$$

The average growth rate over the projection period is

$$\bar{r} = \frac{1}{T}\int_0^T r(t)\,dt ,$$

so even if r(t) is an arbitrary function of time, we can write

$$P_T = P_0 e^{\bar{r}T} .$$

From this it is easy to calculate $\bar{r}$,

$$\bar{r} = \frac{1}{T}\log_e\left(\frac{P_T}{P_0}\right) .$$

The average growth rate, $\bar{r}$, is dimensionless, does not depend on the initial or final population size, and takes the duration of the projection period into account. Since $\bar{r}$ relates the true populations at the beginning and end of the projection period, we call it $\bar{r}_{true}$.

This $\bar{r}_{true}$ sums up in one number the growth of the population over T years. For instance, the population of the United States went from 152 million in 1950 to 213 million in 1975, at an average growth rate of $\log_e(213/152)/25 = .0136$, or 1.36% per year.

The simplest projection method assumes that the population will grow exponentially, as in equation (1), with some value r. We can describe the entire method by one number, call it $\bar{r}_{proj} = r$. For more complex projection methods, we define the average growth rate of the projection as

$$\bar{r}_{proj} = \frac{1}{T}\log_e \frac{\hat{P}_T}{\hat{P}_0} .$$

We use $\hat{P}_0$ rather than P_0 because the true population at time zero may not be known at the time the projection is made and an estimate is used instead.

The Census Bureau's 1950 projection for 1975 was 210 million, based on 152 million in 1950. We calculate $\bar{r}_{proj} = \log_e(210/152)/25 = .0129$, or 1.29% per year.

Finally we define the error term, Δr, as the difference of the average growth rates,

$$\Delta r = 100 \times (\bar{r}_{proj} - \bar{r}_{true}) = \frac{100}{T} \log_e\left(\frac{\hat{P}_T}{\hat{P}_0} \cdot \frac{P_0}{P_T}\right) .$$

The factor of 100 simply makes the numbers more manageable, and reduces them to percentage terms. For the Census Bureau projection, $\Delta r = 100(.0129 - .0136) = -.07$.

The statistic, Δr, summarizes in one number the error in a population projection. It does not depend on the population size, and takes the duration into account. Since we use both $\hat{P}_0$ and P_0, Δr ignores errors caused by a bad estimate of the initial population, or a slight change in the coverage region. In this way we study the method of projection, and the assumptions it makes about growth rates, and not errors in the initial population.

With this new statistic in hand, let us go back to the four projections in Exhibit 1. The smallest Δr, hence the best projection, is -.07 for the U.S. Census Bureau. The largest is for Wigglesworth's 200 year projection. We will see shortly that in terms of Δr, the projection is not particularly bad. But these are only four special cases. To get a better idea about the size of projection errors, we must look at more data.

4. ANALYSIS OF U.S. PROJECTIONS

We first look at some data for the United States. Exhibit 2 presents the Δr for projections made by the U.S. Census Bureau (1946,1953,1956,1962,1966,1971) in "jump off" years 1945 through 1970, for "target" years 1950 through 1975. We present the median projection in all cases. The table is triangular because we can only calculate Δr if the target year has already passed. The -.90 at the lower left means that the 30 year projection from 1945 to 1975 has a Δr of -.90. The -.07 at the top of the

EXHIBIT 2

		JUMP OFF YEAR					
		'45	'50	'55	'60	'65	'70
DURATION	5	-.86	-.07	.02	.11	.11	.32
	10	-.96	-.18	.08	.32	.32	
	15	-1.05	-.20	.26	.52		
	20	-1.03	-.11	.45			
	25	-.97	-.05				
	30	-.90					
AVERAGE		-.96	-.12	.20	.32	.22	.32

Δr FOR U.S. POPULATION PROJECTIONS second column is the Δr for the 1950 projection of the 1975 population, which is described in Exhibit 1. It is among the best predictions in the sample.

Treating the 21 values in Exhibit 2 as a random sample, the average error is -.19 and the standard deviation .54. Compared to these numbers, none of the Δr in Exhibit 1 is out of line, not even Wigglesworth's. Since the average Δr is negative, the projections have been biased downward, that is they have been undershooting the mark. But a closer look reveals a strong pattern in the data. All of the projections made in '45 and '50 were low, and all later projections were high. The average value of Δr for each column appears below Exhibit 2. The message is clear: in '45 and '50, the forecasters did not anticipate the baby boom, and after that they did not realize it would not continue. In the analysis of variance sense the mean values explain over 95% of the variance in Exhibit 2. The standard deviation of the residual Δr, once the means have been removed, is .13, compared to the original .54.

Let us identify the average error for each year as the "jump off bias". This bias partially reflects the fact that the projections were made simultaneously by the same organization, but also reflects something of the attitude of the time among the experts. Dorn (1950) (see Exhibit 3) presents population projections made during the '30's and '40's by Pearl and Reed, Dublin, and the Scripps Institute. Their projections for the United States in 1970 ranged from 145 to 172 million. Since the population turned out to be 205 million, the Δr's for the projections reported by Dorn ranged from -.42 to -1.02.

In order to calculate a confidence interval for a future population, we must first estimate the distribution of the error term, Δr. The previous analysis indicates that there are at least two parts to the error: a bias term which depends on the year of the projection was made, and a random error term. To understand the distribution of Δr, therefore we must study the distributions of both the bias and the random error.

EXHIBIT 3

NAME	YEAR	PROJECTION (IN MILLIONS)	BASE	Δr
PEARL-REED I	'10	167.9	92.4	-.33
PEARL-REED II	'30	160.4	123.0	-.61
DUBLIN	'31	151.0	124.1	-.78
SCRIPPS	'28	171.5	120.5	-.42
SCRIPPS	'31	144.6	124.1	-.89
SCRIPPS	'33	146.0	125.7	-.92
SCRIPPS	'35	155.0	127.4	-.80
SCRIPPS	'43	160.5	136.7	-.90
SCRIPPS	'47	162.0	144.1	-1.02

POPULATION PROJECTIONS FOR U.S. 1970

(ACTUAL POPULATION - 204.9 MILLION)

5. ANALYSIS OF U.N. PROJECTIONS

To get a better idea about the possible size of the bias, we need more data, and turn to the U.N. population projections. They have made projections in 1954, '58, '63, and '68 for the target years '55, '60, '65, '70 and '75. They use the same component method of projection as the U.S. Census Bureau. They divide the world up into 24 regions and make projections for each. The boundaries and number of regions change from time to time, but detailed tables allow one to put together projections for the present 24 regions. We can calculate Δr for 14 of them at this time. The results are in the Appendix.

The first step in the analysis of these data is the calculation of the jump off bias, b_{ij} for each region i and jump off year j, as the mean, over all durations k, of Δr_{ijk}. The residual is then defined as $e_{ijk} = \Delta r_{ijk} - b_{ij}$.

Two stem-and-leaf plots (Tukey 1977) in Exhibit 4 show the distribution of the bias terms for the developed and underdeveloped regions. Stem and leaf plots both preserve the data, and present it for analysis in a form similar to a histogram. The row, or stem, in which a number appears gives the whole number part of the bias term, and the entry in the row, or leaf, gives the first two decimal places. For instance, the "40" circled in Exhibit 4 means a b_{ij} of +.40 for some jump off year in a developed region. The circled "31" indicates a b_{ij} of -1.3 for an underdeveloped region.

The plots in Exhibit 4 compare the location, scale, and shape of the distributions of bias term. The median bias for developed countries is +0.02, almost zero. In the long run, the U.N. Projections for developed regions have been unbiased, although for any given region and jump off year, the bias in Δr ranges from -.91 to +.40. The median bias for underdeveloped regions is -.27 and the range is -1.55 to +.56. Over the years the U.N. has been underestimating future population, and has had larger bias errors for underdeveloped countries. This is undoubtedly due to the scarcity of data for underdeveloped countries.

EXHIBIT 4

DEVELOPED REGIONS

F	(40)
T	20,21,21,22,27,30,34
0·	01,03,03,04,04,05,09,10,16,16,17,18
-0·	00,03,10,13,15,17,17
T	23,23,24,27,27,34
F	41,48
S	
★	91

UNDERDEVELOPED REGIONS

F	46,56
T	20,23,26,27,30,32
0·	01,01,02,06,06,09,10,10,15,15,19
-0·	03,05,08,10,11
T	21,22,22,25,25,26,28,30,30,34,37,39,39
F	42,48,50
S	60,62,67,70,72,72,73,76,78
★	84,87,95,96
-1·	02,02,05
T	21,29,(31)
F	55

JUMP OFF YEAR BIAS

STEM-AND-LEAF PLOT

Exhibit 5 breaks the data down by jump off year. Each "box plot" (Tukey 1977) schematically describes the distribution of b_{ij} for each of the four jump off years. The center horizontal line corresponds to the median of the batch of numbers, and the upper and lower limits of the box correspond to the upper and lower fourths or quartiles of the data. The box therefore represents the central half of the data. We define a point to be an outlier if it is more than 1 1/2 times the length of the box from the nearest fourth. The long vertical lines connect the furthest non-outlying point to the box, and outliers are marked with a heavy dot.

Exhibit 5 indicates that the distribution of the bias for the developed countries has remained relatively stable over the four jump off years. In none of the years has the U.N. been strongly biased, and the magnitude of the error has remained approximately the same. Only in 1968 did they tend to predict larger populations than eventually appeared. In short, as far as bias goes, the earlier projections are about as good as the later ones; the U.N. prediction ability seems to be neither getting better or worse.

We see quite a different picture for the underdeveloped regions. First, the earlier projections were severely biased downward, but the later ones were less severely biased. Second, the variance of the bias term from region to region has not changed drastically over time. An optimistic view is that future U.N. projections will have a distribution of jump off year biases centered around zero, but with the same variance as each of the four years shown in Exhibit 5.

Exhibits 6, 7 and 8 analyze the residuals, after accounting for jump off year bias. The stem-and-leaf plots for developed and underdeveloped regions in Exhibit 6 show a larger residual variance for developed regions. Since bias terms have been subtracted, both distributions are centered at zero.

Exhibit 7 shows box plots for the residual terms broken down by jump off year. By definition, the center must be zero, but the plots show no change in the residual variance from year to year.

EXHIBIT 5

DISTRIBUTION OF BIAS TERM

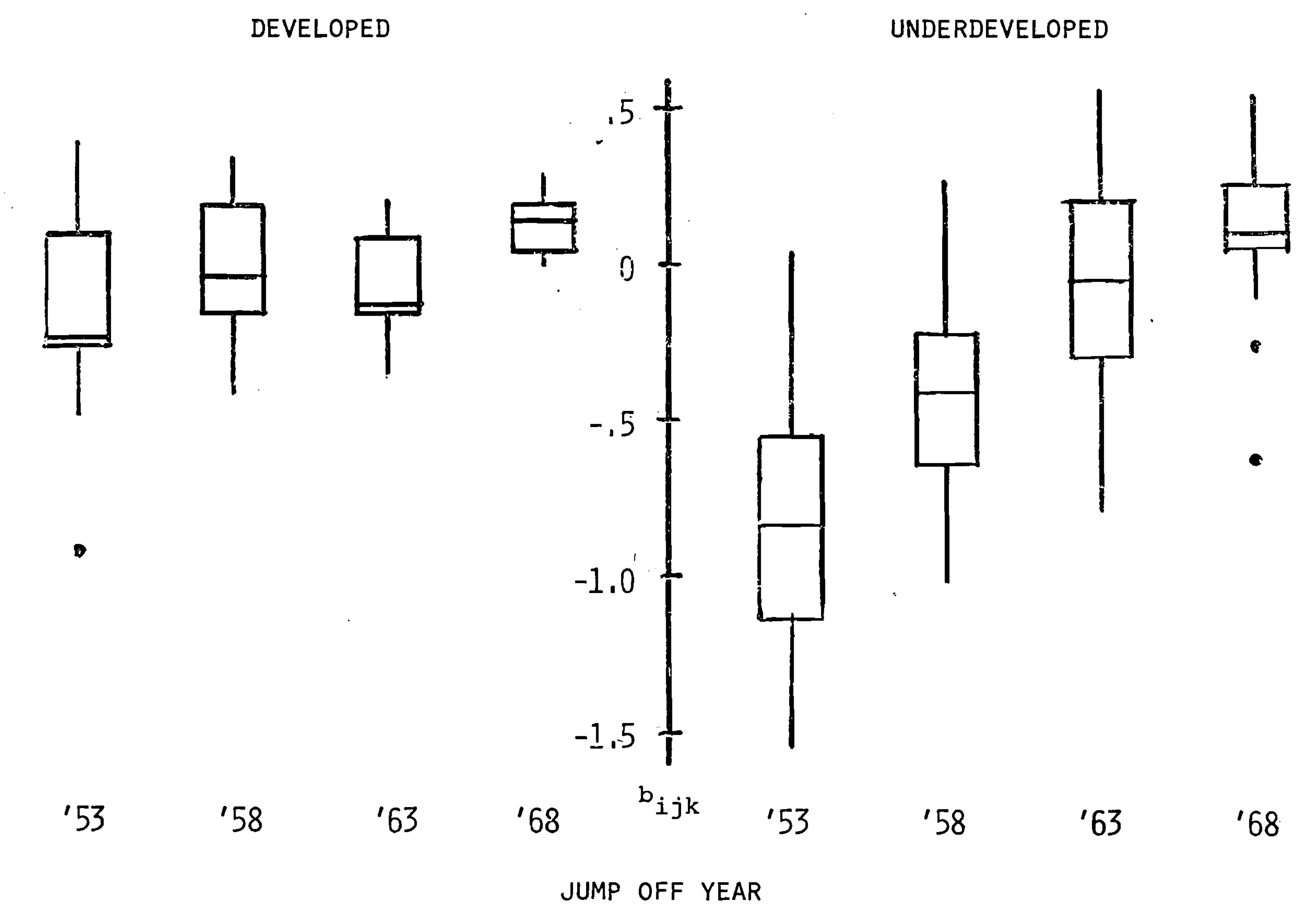


EXHIBIT 6

DEVELOPED REGIONS

```
  ★ | 568
 1' | 01222333
  ★ | 556667888889
 0' | 000000001111111112222222222333333334444444
-0' | 000000001111111112222222233333334444
  ★ | 5555666777888999
-1' | 001223
  ★ | 7
-2' | 0
```

UNDERDEVELOPED REGIONS

```
  ★ | 6          HI   .97,.45
 3' | 2
  ★ | 5
 2' | 1114
  ★ | 899
 1' | 00000122223333344
  ★ | 555556666666677778999
 0' | 000000001111111222222222222223333333333333333444444
-0' | 00000000111111122222222222222333333333333333444444
  ★ | 55555556666666667777788899999999
-1' | 000011222333334
  ★ | 55556666
-2' | 224
  ★ |
-3' | 0
  ★ | 77         LO   -.45
```

RESIDUAL ERROR

STEM-AND-LEAF PLOT

EXHIBIT 7

DISTRIBUTION OF RESIDUAL TERM

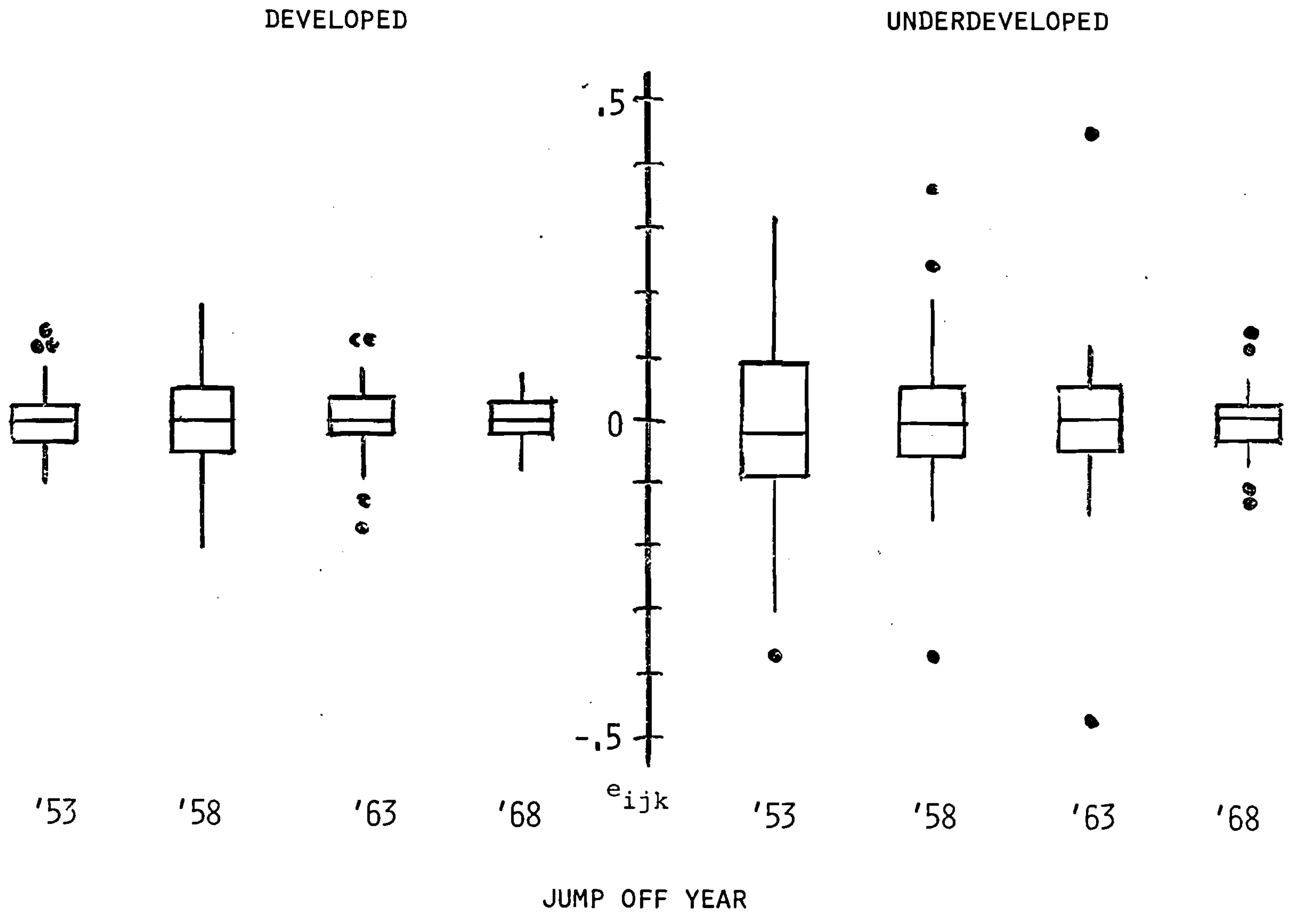


Exhibit 8 breaks the distribution of the residual term down by duration of the projection. There do not seem to be any systematic trends in either the median residual or the residual variance as duration increases. This analysis indicates that Δr, the error in the annual growth rate, has effectively adjusted for the duration effect mentioned in Section 2.

In summary, Δr_{ijk} seems to be made up of two components, a jump-off-year bias, b_{ij} and a random error e_{ijk}. For developed regions the distribution of b_{ij} seems to be stable over time, and centered around zero. For underdeveloped regions, the variance of b_{ij} is stable, but has been centered below zero in the past, although it is centered near zero in the latest projections. The distribution of residuals, on the other hand, is stable over both jump off year and duration. The variance of both the bias and residual distributions is larger for underdeveloped countries.

6. CONFIDENCE INTERVALS FOR U.S. POPULATION PROJECTIONS

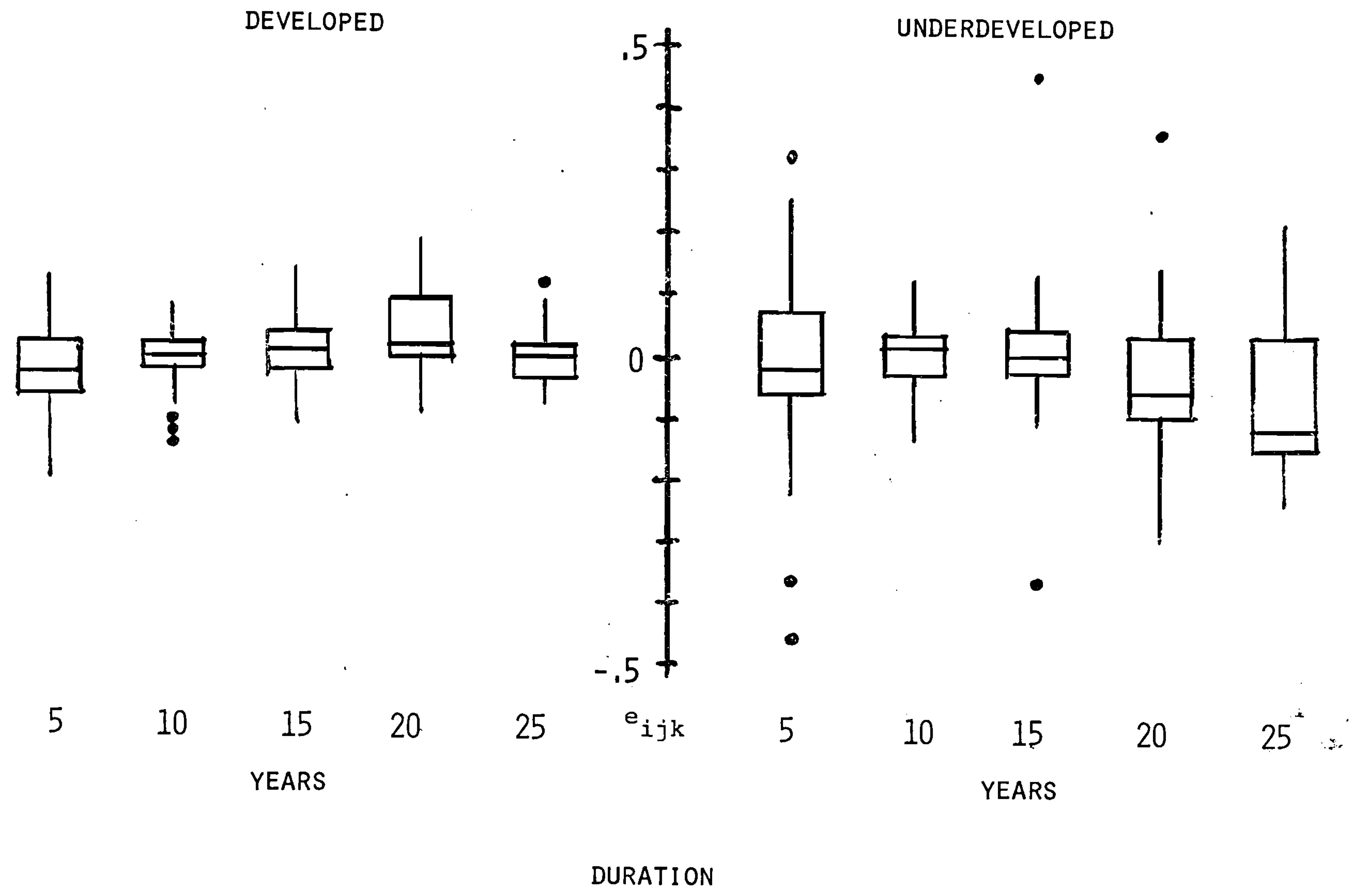
We now return to the original purpose of this paper, the calculation of confidence intervals for population projections. The analysis of the U.S. data shows that two components made up projection errors. The analysis of the U.N. data indicates that at least for the developed countries, the distribution of these terms is relatively stable. Given these conditions, we now use the observed error distributions to infer bounds on Δr, and hence P_T, for the future.

The standard deviation of the 21 values of Δr for the U.S. in Exhibit 2, after the jump off year bias has been removed is .13. The standard deviation of the bias term (based on the five observations) is .50. An estimate of the variance of Δr is then, $Var(\Delta r_{ijj}) = Var(b_{ij}) + Var(e_{ijk}) = .50^2 + .13^2 = .27$, that is the standard deviation of Δr is approximately .52. In other words, the standard deviation of the predicted growth rate is about .52, for a population which has grown at a rate between 1% and 1.5%.

Ideally, to construct confidence intervals for U.S. Census Bureau projections, we would like to consider only U.S. data.

EXHIBIT 8

DISTRIBUTION OF RESIDUAL TERM



But as we have seen, the U.S. data contain only 5 observations on the jump off year bias, not enough to reliably estimate its variance. Instead, we use the error distributions for the U.N. developed regions. That is, lacking enough direct evidence, we consider a larger bank of data for similar regions.

The standard deviation of the bias term for developed regions in Exhibit 4 is .27. That standard deviation of the error term is .08. This yields an estimated standard deviation for Δr of .28. This is about half of the estimate based only on U.S. data, but since it includes a wider experience may more accurately reflect the true variation of Δr.

The two estimates give us an order of magnitude estimate and a range of possibilities for σ, the standard deviation of Δr. We will optimistically use a value of $\sigma = .3$. This means a standard deviation of 0.3% for the projected birth rate.

To construct a confidence interval, we assume that

$$\bar{r}_{true} = \bar{r}_{proj} \pm 2\sigma$$

with probability .95, and

$$\bar{r}_{true} = \bar{r}_{proj} \pm \sigma$$

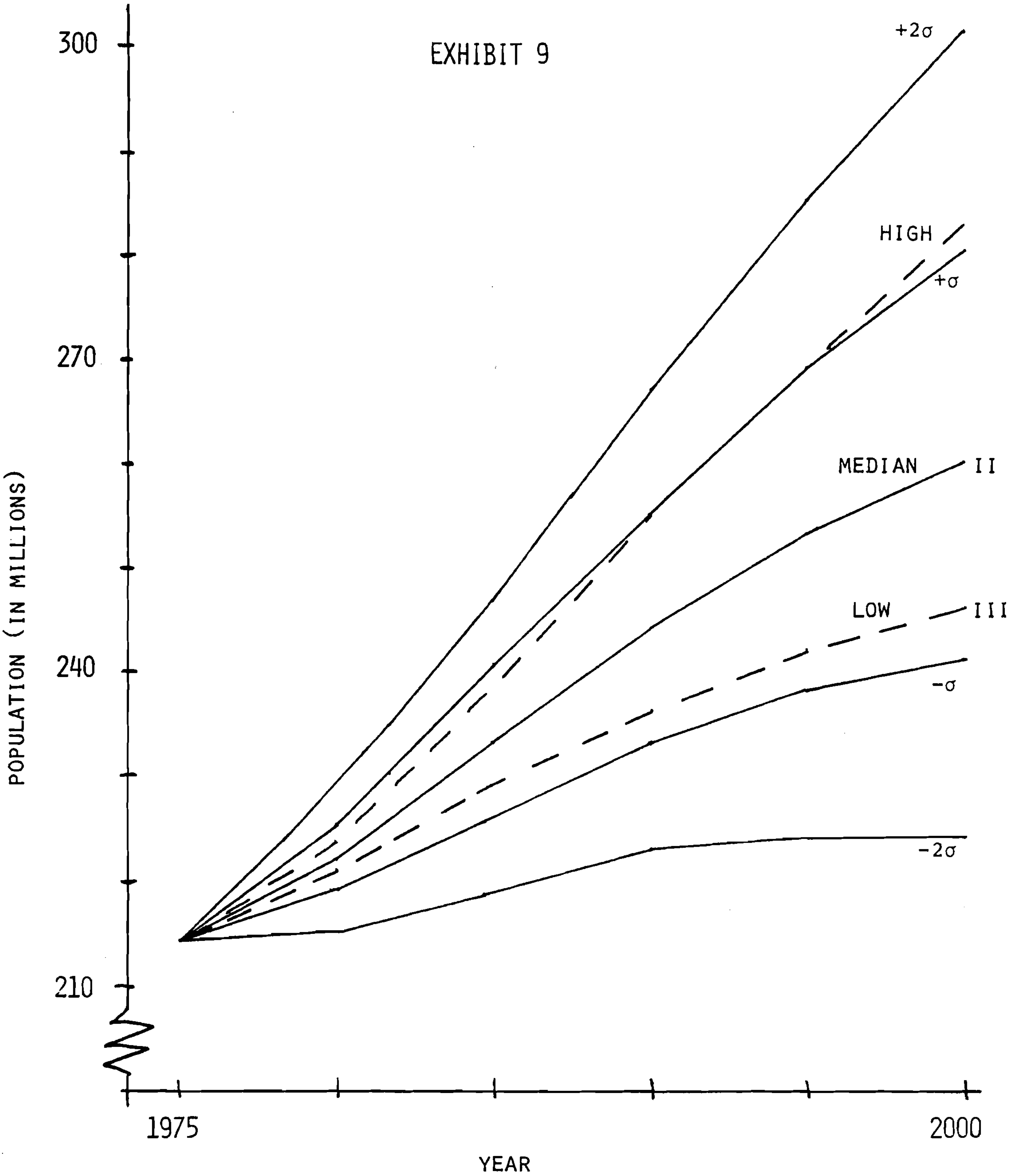
with probability 2/3.

These values would be approximately true if Δr had a Gaussian distribution, and are a good approximation in other cases, especially given the nearly Gaussian shape of the distribution in Exhibits 4 and 6.

Using the relationship

$$P_T = P_0 e^{T\bar{r}_{true}}$$

a 95% confidence interval for P_T is approximately $\left(P_0 e^{T(\bar{r}_{proj} - 2\sigma)}, P_0 e^{T(\bar{r}_{proj} + 2\sigma)}\right)$ and a 2/3 interval $\left(P_0 e^{T(\bar{r}_{proj} - \sigma)}, P_0 e^{T(\bar{r}_{proj} - \sigma)}\right)$. Exhibit 9 plots these intervals for the optimistic estimate $\sigma = .3$.



U.S. CENSUS PROJECTIONS (HIGH, MEDIAN, LOW)
AND σ = .3 CONFIDENCE INTERVALS

For the turn of the century, the 2/3 interval is 241 to 280 million, and the .95% interval is 224 to 302 million. For the purpose of comparison, the U.S. Bureau of the Census' high and low projections are also shown in Exhibit 9. They correspond approximately to the 2/3 interval. A more pessimistic analysis, with $\sigma = .5$ based solely on U.S. data, would give confidence intervals approximately twice as wide.

7. OTHER POPULATION PROJECTIONS METHODS

So far we have examined two very similar sets of projections -- both made by the component method for large scale regions. To gain some perspective we examine in this section two other types of population projection.

Long (1977) presents four sets of population projections for the 50 American states from 1970 to 1975. Two are standard demographic projections made by the U.S. Census Bureau and the National Planning Association. A third is similar to the Census Bureau's projection but assumes no interval migration. The fourth projection, by the U.S. Bureau of Economic Analysis, is based on economic rather than demographic assumptions. The mean value and standard deviation of Δr for each of these sets of projections appears in Exhibit 10. Each set is much more variable than the U.S. or U.N. projections, and they are all, especially the economic projections, seriously biased.

A simple and common population projection technique is to assume that the growth rate over the next T years will be the same as it was over the last T years. This assumption yields the projection formula

$$\hat{P}_T = \frac{P_0}{P_{-T}} \cdot P_0 \quad .$$

The U.N. data allow us to evaluate this technique four times for $T = 5$ and two times for $T = 10$. The mean and standard deviations of the Δr are given in Exhibit 10. For these data, the simple geometric projection technique has been almost unbiased, and has a standard deviation equal to or smaller than the more complicated methods.

EXHIBIT 10

PROJECTION SERIES	BIAS	STANDARD DEVIATION
U.S. CENSUS BUREAU	-.02	.50
U.N. DEVELOPED REGIONS	-.03	.28
U.N. UNDERDEVELOPED REGIONS	-.34	.51
U.S. STATES		
CENSUS I - E	-.23	.92
NATIONAL PLANNING ASSOC.	-.09	.76
U.S. BUREAU OF ECONOMIC ANALYSIS	-.41	.93
CENSUS III - E	-.16	.95
CONSTANT GEOMETRIC GROWTH		
5 YEARS	-.005	.19
10 YEARS	-.01	.32

SUMMARY OF PROJECTION ERRORS

This indicates that the simplest projection method, for some purposes, is better than the more complicated models. Certainly its simplicity and the small amount of data necessary for its application speak in its favor. On the other hand, except for evaluating Wigglesworth's 200 year projection, it has not been adequately tested for durations longer than 10 years. Furthermore, the geometric method only predicts total population size, not age composition, as does the component method. Sometimes, for instance when planning for the Social Security System, it is exactly this age composition that we need. So the interpretation is that for short term, total population projections, simple geometric projection give more accurate results than the more complicated component method.

8. COMPARISON OF PROJECTION TECHNIQUES

Exhibit 10 sums up the evidence we have gathered in this paper about population projections. Population projections for countries or regions tend to have a standard deviation of about .3 or .5 in Δr, that means an error of $\pm 0.3\%$ or $\pm 0.5\%$ per year in growth rates which range from .5% to 2.5% per year. Developed regions are easier to predict than underdeveloped regions. Subnational projections are one half to one third as accurate (in terms of standard deviation) as national or regional projections, and are biased as well. Simple geometric projections have been relatively unbiased and accurate for total population size.

9. PROBLEMS WITH THIS APPROACH

There are three problems with the data-analytical approach of this paper. First, we treat all of the Δr as independent random observations, the actual population sizes from year to year are not independent, and all projections made at one time depend on a common set of assumptions. This error is not serious when talking about the error between two fixed points in time, but from our analysis it is impossible to make simultaneous confidence intervals for two or more future populations. Although more complicated models could handle joint distributions, the amount of arbitrary assumptions needed would be prohibitive.

Second, Δr only analyzes the error in total population size, and not in age composition. Sometimes future age composition, not size, is the main goal of population prediction. But more frequently the total population size is the most important quantity, and the Δr analysis allows us to construct its confidence intervals.

Third, sometimes the aim of a population projection is not for predictive purposes, but to provide a warning about the consequences of present trends. One could argue that these projections are successful only if they are wrong. But Δr is not a measure of success, but simply a measure of the difference between actual and projected populations. It is a measure of the accuracy of projections if, as is commonly done, they are interpreted as predictions.

10. CONCLUSIONS

A historical analysis of certain series of population projections shows that:

1) the yearly growth rate error, Δr, allows an economic and coherent picture of the error structure of population projections;
2) this error, Δr, consists of two factors, a bias associated with the jump off year and a random error term;
3) the distributions of both factors have been relatively stable over time.

The discovery of stable error distributions allows us to transform the results of the historical analysis into confidence intervals for future populations. These confidence intervals reflect the best efforts of competent demographers in the past, and should be a reliable guide to the present generation's ability to predict the future.

The resulting confidence intervals for the U.S. are very large. An optimistic analysis gives a 2/3 confidence interval approximately equal to the Census Bureau's low and high estimates.

A 95% interval for the year 2000 ranges from about 220 to 300 million. State populations are harder to predict accurately. Simple geometric projections of total population for short durations are slightly more accurate.

We do not intend to criticize the construction or use of population projections, for they are clearly necessary planning tools. Nor do we pretend to be able to improve them. Instead we merely attempt to measure their inherent inaccuracy. Hopefully this analysis will enable planners to use projections more objectively by providing a range of reasonable possibilities rather than a single estimate.

REFERENCES

Cannan, E. (1895) The probability of a cessation of the growth of population in England and Wales during the next century. The Economic Journal 5:505-15.

Cohen, J.E. (1976) Ergodicity of age structure in populations with Markovian vital rates. I: Countable states. Journal of the American Statistical Association 71:335-339.

Cohen, J.E. (1977a) Ergodicity of age structure in populations with Markovian vital rates. II: General states. Advances in Appl. Probability 9:18-37.

Cohen, J.E. (1977b) Ergodicity of age structure in populations with Markovian vital rates. III: Finite-state moments and growth rates; illustrations. Advances in Appl. Probability 9:462-475.

Dorn, H.F. (September 1950) Pitfalls in population forecasts and projections. Journal of the American Statistical Association 45:311-34.

Lee, R.D. (1974) Forecasting births in post-transition populations: stochastic renewal with serially correlated fertility. Journal of the American Statistical Association 69:607-617.

Long, J.F. (1977) Prospects for a Composite Demographic-Economic Model of Migration for Subnational Projections, in Report of the Conference on Economic and Demographic Methods for Projecting Population. American Statistical Association.

Sykes, Z.M. (1969) Some stochastic versions of the matrix model for population dynamics. Journal of the American Statistical Association 44:111-30.

Tukey, J.W. (1977) Exploratory Data Analysis. Reading, Mass.: Addison-Wesley.

United Nations (1954) Framework for Future Population Estimates, 1950-1980, by World Regions. Proceedings of the World Population Conference, 1954, Volume III, New York.

United Nations (1958) The Future Growth of World Population. Population Studies 28. ST/SOA/Series A/28, New York.

United Nations (1966) World Population Prospects as Assessed in 1963. Population Studies 41. ST/SOA/Series A/41, New York.

United Nations (1972) World Population Prospects, 1965-2000, as Assessed in 1968. Population Studies 53. ST/SOA/Series A/53, New York.

United Nations (1977) World Population Prospects as Assessed in 1973. Population Studies 60. ST/ESA/Series A/60, New York.

U.S. Bureau of the Census (September 1946) Population Special Reports. Series P-46, No. 7.

U.S. Bureau of the Census (August 1953) Current Population Reports. Series P-25, No. 78.

U.S. Bureau of the Census (November 1958) Current Population Reports. Series P-25, No. 178.

U.S. Bureau of the Census (July 1962) Current Population Reports. Series P-25, No. 251.

U.S. Bureau of the Census (March 1966) Current Population Reports. Series P-25, No. 329.

U.S. Bureau of the Census (November 1971) Current Population Reports. Series P-25, No. 470.

Wigglesworth, E. (1775) Calculations on American Population. Boston.

APPENDIX: Δr FOR U.N. PROJECTIONS

Developed Regions

Japan — Jump off year

Duration	53	58	63	68
5	.30	.37	-.09	.03
10	.48	.39	-.18	-.01
15	.48	.35	-.24	
20	.42	.25		
25	.33			

Western Europe — Jump off year

Duration	53	58	63	68
5	-.10	-.46	-.51	.05
10	-.22	-.54	-.30	.05
15	-.34	-.37	-.21	
20	-.28	-.25		
25	-.23			

Southern Europe — Jump off year

Duration	53	58	63	68
5	.04	-.05	-.15	.22
10	.00	-.02	-.11	.19
15	-.02	.07	-.13	
20	-.00	.12		
25	-.00			

Eastern Europe — Jump off year

Duration	53	58	63	68
5	.11	.25	.24	.22
10	.17	.27	.22	.18
15	.19	.30	.18	
20	.21	.27		
25	.19			

Northern Europe — Jump off year

Duration	53	58	63	68
5	.22	-.18	-.23	.13
10	.13	-.24	-.14	.18
15	.04	-.13	-.13	
20	.06	-.03		
25	.07			

U.S.S.R.

Duration	Jump off year 53	58	63	68
5	-.26	-.02	-.00	.02
10	-.33	.09	.12	.04
15	-.29	.27	.15	
20	-.17	.36		
25	-.11			

North America

Duration	Jump off year 53	58	63	68
5	-.49	-.22	-.08	.08
10	-.55	-.18	.02	.23
15	-.52	-.05	.17	
20	-.45	.05		
25	-.40			

Temperate South America

Duration	Jump off year 53	58	63	68
5	-.25	-.05	.18	.31
10	-.27	-.04	.24	.30
15	-.25	-.04	.23	
20	-.23	.03		
25	-.22			

Australia & New Zealand

Duration	Jump off year 53	58	63	68
5	-.90	-.24	-.28	.00
10	-.91	-.27	-.27	.08
15	-.91	-.24	-.26	
20	-.91	-.32		
25	-.94			

Underdeveloped Regions

China — Jump off year

Duration	53	58	63	68
5	-1.06	.17	-.27	.13
10	-.93	.24	-.31	.07
15	-.83	.30	-.31	
20	-.74	.37		
25	-.63			

Other East Asia — Jump off year

Duration	53	58	63	68
5	.94	-1.32	.19	.23
10	-.13	-1.08	.35	.29
15	-.37	-.82	.41	
20	-.33	-.59		
25	-.25			

Middle South Asia — Jump off year

Duration	53	58	63	68
5	-.51	-.52	-.10	.26
10	-.66	-.55	-.05	.30
15	-.76	-.50	-.00	
20	-.79	-.43		
25	-.78			

South East Asia — Jump off year

Duration	53	58	63	68
5	-.47	-.62	-.05	.15
10	-.67	-.63	-.08	.15
15	-.78	-.60	-.12	
20	-.82	-.56		
25	-.85			

South West Asia — Jump off year

Duration	53	58	63	68
5	-1.27	-.18	-.16	.08
10	-1.25	-.18	-.10	.12
15	-1.21	-.23	-.07	
20	-1.18	-.29		
25	-1.16			

Western Africa — Jump off year

Duration	53	58	63	68
5	-.92	-.74	.27	.09
10	-.98	-.84	.33	.09
15	-1.08	-.89	.29	
20	-1.11	-.99		
25	-1.18			

Eastern Africa — Jump off year

Duration	53	58	63	68
5	-1.11	-.83	-.79	-.10
10	-1.20	-1.00	-.71	-.09
15	-1.31	-1.08	-.68	
20	-1.38	-1.17		
25	-1.45			

Middle Africa — Jump off year

Duration	53	58	63	68
5	-1.10	-.43	-.87	-.36
10	-1.19	-.65	-.80	-.14
15	-1.34	-.78	-.66	
20	-1.45	-.83		
25	-1.47			

Northern Africa — Jump off year

Duration	53	58	63	68
5	-.86	-.53	.22	.16
10	-.94	-.45	.09	.29
15	-.94	-.50	.13	
20	-1.03	-.45		
25	-1.05			

Southern Africa — Jump off year

Duration	53	58	63	68
5	-.22	-.36	-.24	-.74
10	-.24	-.42	-.38	-.49
15	-.34	-.50	-.28	
20	-.46	-.39		
25	-.46			

Tropical South America — Jump off year

Duration	53	58	63	68
5	-.78	-.54	.17	.04
10	-.78	-.43	.20	.08
15	-.76	-.34	.20	
20	-.74	-.25		
25	-.72			

Middle America — Jump off year

Duration	53	58	63	68
5	-.16	-.29	-.06	.21
10	-.31	-.27	.03	.20
15	-.41	-.24	.06	
20	-.46	-.23		
25	-.52			

Caribbean — Jump off year

Duration	53	58	63	68
5	.04	-.09	.10	.42
10	-.01	-.08	.31	.35
15	-.04	.07	.34	
20	.04	.14		
25	.08			

Melanesia — Jump off year

Duration	53	58	63	68
5	-1.23	-.15	-.78	.02
10	-1.43	-.26	-.72	.09
15	-1.60	-.30	-.65	
20	-1.71	-.11		
25	-1.79			

Polynesia — Jump off year

Duration	53	58	63	68
5	-1.04	-.08	.11	.43
10	-.98	-.24	.55	.48
15	-1.05	-.27	1.01	
20	-1.03	-.29		
25	-1.00			