SOURCES OF INDETERMINACY IN VON NEUMANN-
MORGENSTERN UTILITY FUNCTIONS

John C. Hershey
Howard Kunreuther
Paul J.H. Schoemaker

ABSTRACT

Utility functions are an important component of normative decision
analysis. They also serve to characterize the nature of people's risk-taking
attitudes. In this paper we examine various factors that make it difficult
to speak of _the_ utility function for a given person. Similarly we show that
it is questionable to pool data across studies (for descriptive purposes)
that differ in the elicitation methods employed.

The following five sources of indeterminacy are specifically discussed.
First, the certainty equivalence method generally yields more risk-seeking
preferences than the probability equivalence method. Second, the probability
and outcome levels used in reference lotteries induce systematic bias. Third,
combining gain and loss domains yields different utility measures than
separate examinations of the two domains. Fourth, whether a risk is assumed
or transferred away exerts a significant influence on people's preferences in
ways counter to expected utility theory. Finally, context or framing differ-
ences strongly affect choice in a non-normative manner.

The above five factors are first discussed as essential choices to be
made by the decision scientist in constructing Von Neumann-Morgenstern utility
functions. Next, each is examined separately in view of existing literature,
and demonstrated via experiments. The emerging picture is that basic prefer-
ences under uncertainty exhibit serious incompatibilities with traditional
expected utility theory. An important implication of this paper is to
commence development of a systematic theory of utility encoding which incor-
porates the many information processing effects that influence people's
expressed risk preferences.

## INTRODUCTION

The standard model of choice utilized by decision scientists in analyzing problems is expected utility (EU) theory [38]. This model is presumed to be descriptive of people's basic preferences, while having normative implications for more complex problems. Recently, however, there has been an extensive literature which suggests that even basic choice is more complicated than utility theory suggests (see [6] for a review). In view of this, our paper presents a framework for systematically investigating various information processing effects that may confound the elicitation of a decision maker's preferences under uncertainty. The experimental data presented in this study, together with a large body of existing evidence, lead us to the unambiguous conclusion that traditional EU theory needs to be modified if it is to serve as a descriptive and normative model of choice under uncertainty.

Our analysis was, in part, motivated by a recent article of Fishburn and Kochenberger [8] who analyzed 30 empirical utility functions published in earlier literature [32, 12, 9, 10, 3]. These plotted utility functions were either defined on changes in wealth or on return on investment. Fishburn and Kochenberger (F-K) divided each graph into a below-and above-target segment, and fitted linear, power, and exponential functions separately to each subset of data. Of the 30 graphs they examined, 28 were characterized by F-K as having concave (risk-averse) and/or convex (risk-seeking) segments[1], broken down as follows:

|  | Concave Above | Convex Above | Total |
|---|---|---|---|
| Convex Below | 13 | 5 | 18 |
| Concave Below | 3 | 7 | 10 |
|  | 16 | 12 | 28 |

In terms of percentages, 64% of the below-target functions were convex and 57% of the above-target functions were concave. The predominant composite shape, they concluded, was convex-concave (46%) followed by concave-convex (25%).

We question the pooling of utility functions, as was done for instance in the F-K study, when the utility functions are obtained via different elicitation procedures. Specifically, we shall present evidence that the shape of the utility function is influenced by and possibly distorted because of (1) response mode biases, (2) biases induced by probability and outcome levels, (3) aspiration level effects, (4) inertia effects, and (5) context effects. The present paper thus raises a set of methodological issues that have significant implications for both descriptive and prescriptive analyses of choice under uncertainty.

## ELICITATION METHODS

To begin our analysis, we assume that Von Neumann-Morgenstern utility functions [38] are constructed via standard reference lotteries where the client provides indifference judgments between a sure option and a two-outcome lottery. In conducting the elicitation interview, the decision analyst will thus present the client with the following choice:

$$S \quad \text{versus} \quad \begin{array}{c} p \nearrow G \\ \diagdown \\ 1-p \searrow L \end{array}$$
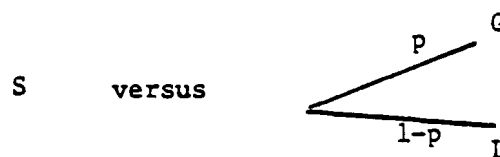
where S is the sure amount, p is the probability of winning G (for gain), and L (for loss) the lower outcome of the lottery. Of course, $0 < p < 1$ and

L < S < G. Note that L and G refer to relative rather than absolute amounts; hence they are not constrained sign-wise. Of these four variables, three will have been set by the decision analyst, whereas the fourth is varied to obtain an indifference judgment such that U(S) = pU(G) + (1-p)U(L). Hence, there exist essentially four different methods for constructing NM utility functions, namely:

1. The certainty equivalence (CE) method, where the client states an indifference level for S for given values of p, G and L.

2. The probability equivalence (PE) method, where an indifference level for p is elicited, for given values of G, L and S.

3. The gain equivalence (GE) method, where the probabilistic outcome G is elicited, and p, L and S are fixed.

4. The loss equivalence (LE) method, where the probabilistic outcome L is elicited, while p, G and S are held constant.

Hence, one important choice the decision analyst must make is which of these four response modes to use. The most common ones are the CE and PE methods. As we shall show, however, there may exist significant differences in risk-taking attitude between these two methods. This, of course, is counter to EU theory.

Another important decision involves the dimensions of the lottery. Specifically, what probability and outcome levels should one use in eliciting risk preferences? If the shape of the utility function depends on the end-points associated with G and L magnitudes, and/or the values of p utilized, we must be aware of this in designing a set of reference lotteries. Again, in theory the choice of levels is arbitrary. Due to the substitution and other axioms of utility theory, an NM utility function constructed with 50-50

reference lotteries should assume the same shape as one obtained with, for example, 30 - 70 lotteries. As we will see, however, this may not be the case due to probability distortions.

A third decision to be made by the analyst concerns the domain of outcomes to be used. Three lottery types may be distinguished, namely pure loss lotteries ($L < G \leq 0$), mixed lotteries ($L < 0$ and $G > 0$), and pure gain lotteries ($G > L \geq 0$). Of course, within the EU model it is arbitrary which approach is used, as the same functional shape (within positive linear transformations) should occur. Hence, an NM function constructed on [-\$1000, \$1000] using mixed lotteries should be identical to one using pure lotteries within the positive and negative subintervals of that range. In practice, however, the functions may well differ (as we shall show), due to aspiration level and possibly other factors.

A fourth decision to be made is how to present the choice to the decision maker; will it be one where the client must assume risk or one where risk is transferred away? For instance, the decision analyst might ask for how much (at a minimum) the client would sell a given lottery(i.e., transfer risk). Alternatively, it might be asked whether the client would exchange a sure gift for that lottery (i.e., assume the risk), which may be quite different psychologically from a transfer of risk, due to inertia effects.

Finally, the decision analyst must choose a decision context for the reference lotteries used. This aspect of the elicitation procedure is important as different wordings, scripts, or scenarios may lead to different stated risk preferences. If the underlying choices are structurally the same, such contextual differences should be without effects. However, since different contexts often emphasize different aspects [1], people may process information

differently, thereby inducing inconsistent responses.

In Fig. 1 we diagram the five types of choices the analyst must make (either implicitly or explicitly). In the remainder of the paper we will demonstrate that each of these five choices may indeed influence the utility function in non-normative ways. As such, we view this paper as a first step in the development of a much needed theory for utility encoding. Compared to probability encoding [31], the value side has largely been ignored in decision analysis although it similarly suffers from serious, systematic biases.

## RESPONSE MODE BIAS

In Table 1 we have summarized which methods were used in each of the five studies examined by Fishburn and Kochenberger [8], together with their findings. Interestingly, for those studies [32, 12, 3] using the certainty equivalent (CE) method, 16 of the 17 below-target shapes were convex and 13 of the 17 above-target shapes were concave, whereas for those studies [9, 10] using the probability equivalence method, 9 of the 11 below-target shapes were concave and 8 of the 11 above-target shapes convex. (Note that none of these studies employed the GE or LE methods.) Hence, there appears to be a strong interaction between the elicitation method used and the predominant shapes obtained by F-K as shown in the following cross-classification derived from Table 1.

|  | Response Mode | |
| --- | --- | --- |
| Composite Shape | Certainty Equivalence | Probability Equivalence |
| Convex Below-<br>Concave Above | 12 | 1 |
| Concave Below-<br>Convex Above | 0 | 7 |

FIGURE 1

CHOICES FOR SELECTING AN ELICITATION PROCEDURE

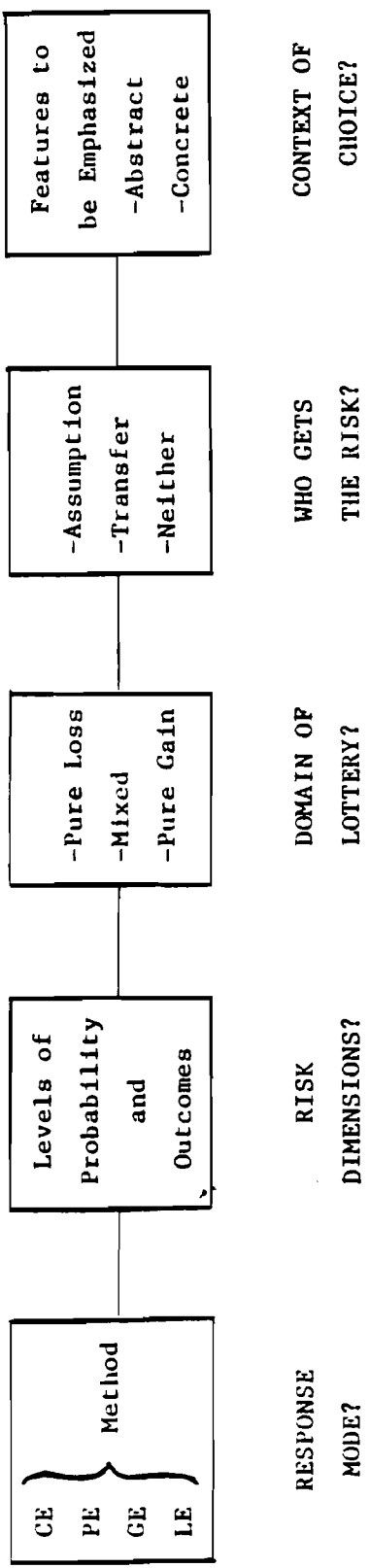| CE <br> PE <br> GE <br> LE } Method | Levels of <br> Probability <br> and <br> Outcomes | -Pure Loss <br> -Mixed <br> -Pure Gain | -Assumption <br> -Transfer <br> -Neither | Features to <br> be Emphasized <br> -Abstract <br> -Concrete |
|---|---|---|---|---|
| RESPONSE <br> MODE? | RISK <br> DIMENSIONS? | DOMAIN OF <br> LOTTERY? | WHO GETS <br> THE RISK? | CONTEXT OF <br> CHOICE? |

TABLE 1

MEASUREMENT METHODOLOGIES AND COMPOSITE SHAPES FOR
UTILITY FUNCTIONS DRAWN FROM FIVE SOURCES

| Source | Certainty Equivalent | Probability Equivalent | Gain and Loss Equivalents | Composite Shape (above and below) | | | |
|---|---|---|---|---|---|---|---|
| | | | | Concave Convex | Convex Convex | Concave Concave | Convex Concave |
| Swalm [32] | Elicited | Constant (at .5) | Variable | 10 | 2 | 1 | 0 |
| Halter and Dean [12] | Elicited | Variable (.1, .2, ..., .9) | Constant (-$50,000, +$100,000) | 2 | 0 | 0 | 0 |
| Grayson [9] | Constant (at $0) | Elicited | Variable | 1 | 1 | 2 | 4 |
| Green [10] | Constant (at $0) | Elicited | Variable | 0 | 0 | 0 | 3 |
| Barnes and Reinmuth [3] | Elicited | Constant (at .5) | Variable | 0 | 2 | 0 | 0 |
| All Studies Combined | | | | 13 | 5 | 3 | 7 |

The conclusion by F-K that the predominant composite shape is convex-concave may thus be artifactual, reflecting instead the higher incidence of the certainty equivalence method in the studies examined.  Indeed, differences in elicitation method account for almost all of the variance among the two most predominant shapes.

### Experiment 1

To examine this response mode bias more systematically, we conducted an experiment with 64 Wharton students taking an undergraduate course in decision sciences.  All students were familiar with NM utility theory, both from lectures and reading Swalm [32].  The subjects were randomly assigned to one of two groups.  The first group was given ten simple questions using the certainty equivalence (CE) method; the other group received the same questions using the probability equivalence (PE) method.  Each group was introduced to its particular method through an example.  Thereafter, the subjects were presented with a choice between a simple gamble and a sure amount of equal expected value.  Subjects in the CE group were asked to increase or decrease the sure amount (depending on their preference), until they were indifferent between the sure amount and the gamble.  Each subject in the PE group was asked to revise the probability of loss in the reference gamble until he or she was indifferent between the gamble and the sure amount.[2]

All ten questions involved pure losses, as shown in Table 2 (see the columns on the left).  The middle columns list the percentages of subjects who were risk averse, indifferent, or risk seeking under the CE and PE methods.  In the last column, chi-squares (and their significance levels) are listed to  indicate whether the response mode used affected the distribution

TABLE 2

RISK-TAKING PATTERNS UNDER CE AND PE METHODS (EXP. 1)

| Question | Sure Loss | Gamble p | Gamble L | Certainty Equivalence Method (N = 32) Risk Averse | Indifferent | Risk Seeking | Probability Equivalence Method (N = 32) Risk Averse | Indifferent | Risk Seeking | Chi-Square |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $ 100 | .5 | $ 200 | 6% | 28% | 66% | 32% | 23% | 45% | 6.97* |
| 2 | 900 | .9 | 1000 | 9 | 13 | 78 | 44 | 13 | 44 | 10.22** |
| 3 | 100 | .05 | 2000 | 19 | 13 | 69 | 41 | 13 | 47 | 3.90 |
| 4 | 10 | .5 | 20 | 0 | 13 | 88 | 6 | 38 | 56 | 8.17* |
| 5 | 100 | .1 | 1000 | 16 | 28 | 56 | 32 | 13 | 55 | 3.60 |
| 6 | 90 | .9 | 100 | 3 | 16 | 82 | 35 | 29 | 35 | 15.55** |
| 7 | 1900 | .95 | 2000 | 19 | 31 | 50 | 47 | 25 | 28 | 6.04* |
| 8 | 10 | .05 | 200 | 34 | 22 | 44 | 31 | 13 | 56 | 1.37 |
| 9 | 10 | .1 | 100 | 22 | 28 | 50 | 38 | 22 | 41 | 1.88 |
| 10 | 190 | .95 | 200 | 3 | 31 | 66 | 38 | 22 | 41 | 11.72** |

NOTE: Asterisks denote significant interaction at the .05 (*) or .01 (**) level. Percentages may not total 100% due to rounding.

of preferences (i.e., for each question, a 2 x 3 contingency table was constructed).

For six of the ten questions, the subjects' preferences (measured simply by type of risk-taking attitude) were influenced by the elicitation method used. As shown in Table 2 the impact of different elicitation methods on outcomes appears strongest for gambles where there is a relatively high probability of losing and weakest when it is relatively low.

It is telling as well to examine how much risk-taking occurred under either procedure when combining all ten questions. In the CE group 29 of the 32 subjects gave risk seeking responses for a majority of the questions, as compared with only 16 in the PE group. This difference is statistically significant (p < .01), and in the same direction as in Tables 1 and 2, i.e., the CE method leads to more risk seeking than the PE method.

In a recent experiment by Wehrung et al. [39] similar response mode biases were observed. They examined around 90 executives using both the certainty equivalence and gain equivalence (GE) methods. Although these two methods generally yielded significant differences (within the same person), Wehrung et al. did not find a systematic bias toward risk-aversion for a given method (as we did). One reason could be that both of their methods entailed payoff (as opposed to probability) adjustments. Another might be that Wehrung et al.'s GE questions used rate of return responses, whereas their CE questions focused on after-tax net profit. Such a contextual difference, as we shall see later, may be a confounding factor.

## RISK DIMENSIONS

Let us now turn to the role of probability and payoff levels. A large body of experimental literature (see Schoemaker [26] for a review) suggests

that people have difficulty combining information from different dimensions, including the multiplication of probabilities and dollar amounts. Such information processing limitations are, for instance, evident from the appeal and predictive power of various types of non-compensatory choice models (e.g., disjunctive and lexicographic models). One recent example is the preference tree model by Tversky and Sattath [35], which describes choice as a covert hierarchical elimination process. A similar approach is taken in Bettman's [4] information processing theory of consumer choice.

In general it seems that people often focus on one dimension at a time, for example, probabilities or outcomes. Under such a model one would not expect the type of utility curves proposed by F-K. Indeed, Laughhunn et al. [18] found, in a study of 224 managers, a strong tendency toward risk-seeking in the loss domain except for very large losses. When the possibility of bankruptcy was introduced, the managers actually became risk-averse, in spite of the low probability of bankruptcy. Not always, however, will the focus shift this way. For instance, Kunreuther [17] showed that consumers may ignore low probability events with catastrophic losses if the probability falls below a certain threshold.

Such single dimension focus and shifting of attention as a function of the levels of the dimensions is not compatible with the below-target risk-seeking hypothesis advanced by F-K. For instance, a recent study by Hershey and Schoemaker [13] showed that the utility function over losses cannot be characterized as being either purely convex or purely concave. This study of basic risk-taking attitudes toward losses involved 18 questions, where the choice for each was between a sure loss S and a probabilistic loss L with a probability p of occurring. To account for some results within the EU model,

Hershey and Schoemaker proposed a Markowitz [21] type utility function which is risk-averse for small losses and risk-seeking for large ones. Even this utility function, however, did not fully explain the empirical data, and hence it was further proposed that people distort probabilities in the manner suggested in prospect theory [15] (i.e., overweighting of low probabilities and underweighting of high ones). These conclusions, however, only concern the loss side. To similarly examine the gain side, particularly the risk-aversion hypothesis, we present the following data.

### Experiment 2

As part of the Hershey-Schoemaker experiment reported in [13], data were collected as well on the gain side (although not reported). The subjects were 82 Wharton MBA students taking an introductory quantitative methods course.[3] These students were presented with the 18 binary choices shown in Table 3. (Note that all are actuarially fair.)

The first six questions provide evidence of considerable risk-seeking on the gain side, particularly for small amounts. Analyses of within-subject preferences, for instance, showed that 57% of the subjects were willing to gamble for a majority of the questions, with only 23% preferring the sure amount for a majority of the questions. (The remaining subjects had exactly three risk-averse and three risk-seeking responses.) In total, 34% were willing to gamble for all six questions, while only 10% chose the sure amount for all six questions. The percentage differences in both of these percentage comparisons are statistically significant at the .01 level.

In the next set of seven questions, the potential gain G was fixed at $10,000 while the probability of winning was varied from .001 to .999. For these questions, risk-aversion increases as the sure amount (S) increases.

TABLE 3

HOW RISK-AVERSE ARE SUBJECTS FOR GAINS?   (EXP. 2)

| Question | Gamble | | Sure Amount | Percent Risk-Averse |
|---|---|---|---|---|
| | p | G | S | (N=82) |
| 1 | .001 | $ 10,000 | $ 10 | 47.6% |
| 2 | .005 | 2,000 | 10 | 41.5 |
| 3 | .01 | 1,000 | 10 | 39.0 |
| 4 | .05 | 200 | 10 | 25.6 |
| 5 | .10 | 100 | 10 | 23.2 |
| 6 | .20 | 50 | 10 | 31.7 |
| 7 | .001 | 10,000 | 10 | 50.0 |
| 8 | .01 | 10,000 | 100 | 54.9 |
| 9 | .10 | 10,000 | 1,000 | 69.5 |
| 10 | .50 | 10,000 | 5,000 | 74.4 |
| 11 | .90 | 10,000 | 9,000 | 78.0 |
| 12 | .99 | 10,000 | 9,900 | 70.7 |
| 13 | .999 | 10,000 | 9,990 | 74.4 |
| 14 | .01 | 100 | 1 | 15.9 |
| 15 | .01 | 1,000 | 10 | 35.4 |
| 16 | .01 | 10,000 | 100 | 59.8 |
| 17 | .01 | 100,000 | 1,000 | 69.5 |
| 18 | .01 | 1,000,000 | 10,000 | 80.5 |

Within-subject analyses reveal that 83% chose the safe alternative for a
majority of the seven questions. For questions fourteen through eighteen,
the probability of gain was fixed at .01 while the potential gain increased
from $100 to $1,000,000. Only 16% were risk-averse for question fourteen,
but this preference increased steadily to the point where 81% were risk-averse
for question eighteen. Within-subject analyses indicate that 85% of those who
preferred the risky alternative for question fourteen had switched preferences
by question eighteen.

Taken together, these results indicate considerable risk-seeking for
gains, particularly for small amounts and low probabilities. The empirical
findings could be explained by (1) a convex portion in the utility curve for
gains, (2) an overweighting of low probabilities, or (3) a combination of the
two. Some recent pilot experiments, however, make explanation (1) unlikely.
For instance, the vast majority of the subjects tested preferred $160 for sure
over a .8 chance at $200 (which is contrary to the convex utility curve
explanation). This leaves explanations (2) and (3) which both include proba-
bility distortions.

The nature of this bias makes it particularly difficult to speak
of _the_ utility function for an individual. For example, if the function is
elicited with 50-50 lotteries, it may be shaped differently from when it is
based on 70-30 lotteries. Empirical evidence for such probability dependency
of the utility curve was offered by Van Dam [36] and Karmarker [16], who
found systematic relationships between the degree of risk-aversion and the
type of odds used in the reference lotteries. Officer and Halter [23]
encountered similar difficulties when using indifference probabilities for
the construction of $U(x)$. It merits further research to learn to what extent

probability biases reflect differential dimension focus and/or probability thresholds. Risk-taking attitudes, however, cannot be generally characterized as purely risk-seeking for losses and risk-averse for gains, even though this may be a fair statement within certain probability and outcome ranges.[4]

## DOMAIN OF LOTTERIES

When eliciting preferences to derive NM utility functions, the decision scientist can use pure loss lotteries ($L < G \leq 0$), mixed lotteries ($L < 0$ and $G > 0$), and/or pure gain lotteries ($G > L \geq 0$). Earlier studies, however, suggest that mixed lotteries induce a much greater degree of risk-aversion than do choices under a pure loss situation. For example, Williams [34] found that translations of pure loss outcomes into a mixture of losses and gains (by adding a fixed amount to all outcomes) produced a dramatic shift from risk-seeking to risk-aversion.

Payne et al. [24] found this same result for three outcome lotteries. They further showed that additional translations into pure gain lotteries induced yet greater risk-aversion. One suggested explanation for this "domain bias" centers on the role of aspiration levels. Such target or reference points are formally incorporated in Fishburn's $\alpha$-t model [7], and prospect theory [15]. These recent models are supported by Payne et al.'s [24] finding that preference reversals within pairs of mixed lotteries are most pronounced when the translations are such that one lottery has only positive outcomes or only negative outcomes, while the other has mixed ones.

Interestingly, the effect of the lottery domain is discernible as well from the five studies examined by F-K. The Grayson [9], Green [10] and Halter-Dean [12] studies employed mixed lotteries (i.e., losses combined with

gains) the majority of which were assessed relative to the status quo.[5] The other two studies [3, 32], on the other hand, used predominantly pure gain and pure loss lotteries. The following simple cross-classification of the F-K data show the importance of this difference.

| Gamble Used | Loss Side Convex | Loss Side Concave | Gain Side Convex | Gain Side Concave | Total Number of Curves |
|---|---|---|---|---|---|
| Pure | 14 | 1 | 4 | 11 | 15 |
| Mixed | 4 | 9 | 8 | 5 | 13 |

$$(\chi_1^2 = 11.8; \ p < .001) \quad (\chi_1^2 = 3.5; \ p < .07)$$

On the loss side, the pure gamble methods yielded 14 out of 15 risk-seeking curves, compared with only 4 out of 13 when mixed gambles were used. On the gain side, however, the effect was reversed, i.e., the pure lottery method yielded greater risk-aversion (11 out of 15) than the mixed gamble studies (5 out of 13).[6] As shown below the cross-classifications, these differences are both statistically significant under a chi-square test, being somewhat weaker on the gain side.

Unfortunately, the above results are confounded with a response-mode effect, since, except for the Halter and Dean study, those experiments using the CE method employed pure lotteries whereas those using the PE method utilized mixed lotteries. In the following experiment, we therefore examine translations from pure loss lotteries to mixed lotteries while holding the response mode constant.

## Experiment 3

The experiment was conducted with 26 undergraduate Wharton students who were taking an introductory computer programming course for managerial

applications. These students were generally unfamiliar with NM utility
theory. Using a within-subject design, each student received two consecutive
questionnaires separated by one week. One of these questionnaires contained
only pure loss choices, identical to those shown earlier in Table 2. The
questions asked for certainty equivalence judgments, using a context of
risk transfer. In the other questionnaire the same ten gambles were used
except that all had been translated into mixed gambles. The amounts added
were such that each mixed gamble had an expected value of exactly zero.
For instance, question one in Table 2 was translated into a 50-50 chance of
winning $100 or losing $100. Note that the order of the two questionnaires
was randomized. No order effects were found. Also, an additional 45
subjects answered only the loss questionnaire and not the mixed one, whereas
nine other subjects answered only the mixed questionnaire. Thus, a total of
71 subjects completed the loss questionnaire, 35 the mixed one, and 26 both.
Table 4 therefore offers a between-as well as within-subject analysis.

As Table 4 demonstrates, positive translations from pure to mixed lotter-
ies significantly increased the percentage of risk-averse responses. To
assess the impact of these translations on all three types of risk-attitudes
(i.e., averse, neutral and seeking), two by three chi-square tables were
analyzed where the rows represent questionnaire type. For all ten questions,
the positive translations significantly ($p < .05$) affected risk-taking
preferences.

The last three columns of Table 4 provide within-subject findings for
those 26 subjects who completed both questionnaires. These results show that
subjects were significantly more risk-averse for mixed lottery questions than
for pure loss lottery questions. For instance, for question one, 14 subjects

TABLE 4

THE EFFECT OF DOMAIN SHIFTS (EXP. 3)

| Question | Pure Loss Questions (N = 71) | | | Mixed Gambles Questions (N = 35) | | | Chi-Square for Translation Effect | Within-Subject Results (N = 26) | | Statistical Significance of Difference |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Number of Subjects Being More Risk-Averse With | | |
| | Averse | Neutral | Seeking | Averse | Neutral | Seeking | | Loss Q. | Mixed Q. | |
| 1 | 11% | 21% | 67% | 53% | 26% | 21% | 25.63** | 14 | 2 | .002 |
| 2 | 23 | 32 | 45 | 77 | 14 | 9 | 29.61** | 17 | 1 | .000 |
| 3 | 24 | 20 | 56 | 74 | 14 | 11 | 26.42** | 17 | 3 | .001 |
| 4 | 6 | 21 | 73 | 26 | 34 | 40 | 14.06** | 14 | 3 | .006 |
| 5 | 17 | 27 | 56 | 71 | 20 | 9 | 33.58** | 19 | 2 | .000 |
| 6 | 18 | 24 | 58 | 46 | 14 | 40 | 8.90* | 10 | 5 | .15 |
| 7 | 21 | 28 | 51 | 60 | 20 | 20 | 16.49** | 13 | 2 | .004 |
| 8 | 39 | 24 | 37 | 66 | 29 | 6 | 12.05** | 8 | 4 | .19 |
| 9 | 27 | 28 | 45 | 71 | 26 | 3 | 24.74** | 11 | 5 | .11 |
| 10 | 15 | 32 | 52 | 40 | 26 | 34 | 7.94* | 11 | 4 | .06 |

NOTE: Asterisks denote significant interaction at the .05 (*) or .01 (**) level. Within-subject significance levels were derived from a one-tailed binomial test. See Table 2 for the levels of the pure loss questions.

were more risk-averse under the mixed than the pure loss question, with only
two subjects being more risk-seeking. This difference is highly significant
(p < .002) under a binomial test, as are most of the other differences.

The above experiment confirms that translations of lottery domains lead
to the predicted shifts in preference, even when holding response mode
constant. Such preference shifts are inconsistent with a utility function
that is uniformly concave, although they may be consistent with a utility
function which is convex for losses and concave for gains.[7] An intriguing
question meriting further research is whether the measured shape <u>within</u>
the loss or gain domain itself is influenced by the lottery domains chosen.
Such a result would, of course, be inconsistent with NM utility theory.

## TRANSFER VS. ASSUMPTION OF RISK

An important choice the decision scientist must make is whether to place
the client in a position of transferring risk or assuming risk (or neither).
With pure loss lotteries, for example, under the CE method, the client is
typically asked to specify a sure payment such that he or she would be indif-
ferent between retaining the pure loss lottery and transferring it. The
indifference premium for purchasing insurance protection is a natural example
of this. On the other hand, with mixed lotteries, when using the CE response
mode, the client is typically asked to specify a sure receipt or purchase
price representing the certainty equivalent for assuming the mixed lottery.
A natural example is the reservation price for entering into a speculative
venture (e.g., oil drilling). These two types of questions differ not only as
to the domain of the lotteries employed, but also in the sense that one ques-
tion involves a transfer of risk while the other involves assumption of risk.

This latter difference may lead to an "inertia" (or "status quo") bias.

The inertia bias concerns a tendency of people to prefer their current wealth position unless presented with clearly superior alternatives (after including transactions costs). It may thus be much easier to convince a person to retain a given risk (when already part of the psychological status quo) than to assume that risk. A convincing example of this is offered in Thaler [33] who compared the prices people would pay to protect against a low probability lethal illness vs. the compensation they would require to expose themselves voluntarily (e.g., for medical research) to this illness. Typically, the mean responses differ by a factor of ten or more, ruling out income effects or transaction costs as a likely explanation. Williams [40] was one of the first to propose the inertia hypothesis for gambles. Since his experiments only showed that subjects were more willing to retain pure risks than assume speculative risks, he proposed that future experiments control for the separate effects of inertia and lottery domain. In the following experiment we examine the effect of transfer vs. assumption of risk for otherwise identical mixed lotteries.

### Experiment 4

The subjects for this experiment were similar to those of experiment 3. The mixed lottery questionnaire in that experiment, which was given to 35 students, contained 10 questions with the following wording (the example refers to the first question).

> You can choose to be in a situation where there is a 50% chance of winning $100 and a 50% chance of losing $100. Would you choose to be in this situation if it did not cost you anything?
>
> YES NO INDIFFERENT

In Table 4 we showed the subjects' risk-preferences for this and the other nine questions to demonstrate the effect of translating a gamble through the origin. To test for a possible inertia bias, 33 other subjects were presented with the same questions worded as follows:

> You are in a situation where you have a 50% chance of winning $100 and a 50% chance of losing $100. Would you be willing to transfer this risk to another person at no cost to you?
>
> YES       NO       INDIFFERENT

Since this question is identical in underlying structure to the earlier one, EU theory would not predict a significant difference in the precentages of risk-averse, indifferent, and risk-seeking responses.[8]

In Table 5, we compare subjects' responses between these two types of wording, i.e., transfer vs. assumption of risk. For reference, the left side of Table 5 shows the specific mixed lottery corresponding to each question. The middle columns indicate the percentages who were risk-averse, indifferent, and risk-seeking under each wording. The final column contains the chi-square values and significance levels. The results show that there is a significant bias, in the expected direction, for three of the 10 questions.

Comparing Tables 4 and 5, it can be seen that in experiment four the inertia effect may have confounded the domain bias results of the third experiment. Together, however, the experiments establish the independent existence of both a significant inertia bias and a significant domain bias.

The inertia effect is strongest for lotteries offering a large probability of a small gain and a small probability of a large loss. One explanation is that such gambles are likely candidates for threshold and certainty

TABLE 5

TRANSFER vs. ASSUMPTION OF RISK (EXP. 4)

| Question | Probabilistic Loss | | Probabilistic Gain | | Assumption of Risk (N = 35) | | | Transfer of Risk (N = 33) | | | Chi-Square |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | L | P(L) | G | P(G) | Risk Averse | Indifferent | Risk Seeking | Risk Averse | Indifferent | Risk Seeking | |
| 1 | $ 100 | .5 | $ 100 | .5 | 53% | 26% | 21% | 48% | 42% | 9% | 2.80 |
| 2 | 100 | .9 | 900 | .1 | 77 | 14 | 9 | 55 | 24 | 21 | 4.04 |
| 3 | 1900 | .05 | 100 | .95 | 74 | 14 | 11 | 52 | 27 | 21 | 3.79 |
| 4 | 10 | .5 | 10 | .5 | 26 | 34 | 40 | 33 | 39 | 27 | 1.27 |
| 5 | 900 | .1 | 100 | .9 | 71 | 20 | 9 | 39 | 33 | 27 | 7.63* |
| 6 | 10 | .9 | 90 | .1 | 46 | 14 | 40 | 48 | 24 | 27 | 1.72 |
| 7 | 100 | .95 | 1900 | .05 | 60 | 20 | 20 | 64 | 15 | 21 | .27 |
| 8 | 190 | .05 | 10 | .95 | 66 | 29 | 6 | 42 | 27 | 30 | 7.54* |
| 9 | 90 | .1 | 10 | .9 | 71 | 26 | 3 | 30 | 36 | 33 | 15.17** |
| 10 | 10 | .95 | 190 | .05 | 40 | 26 | 34 | 36 | 30 | 33 | .19 |

NOTE: Asterisks denote significant interaction at the .05 (*) or .01 (**) level.

effects (regarding the probability dimension), making them more of a sure option (as coded psychologically) than the other lotteries. This explanation, of course, assumes that the inertia effect will be stronger for relatively certain options than very iffy alternatives. This conjecture, however, would need independent verification.

## CONTEXT EFFECT

The above inertia effect is a special type of context effect. In general we shall define context effects as influences on preferences that are without normative basis. As shown, the way information is presented to individuals regarding choices under uncertainty affects their final choice. Further evidence supporting this point comes from a number of studies that are summarized in Tversky and Kahneman [34]. Their principal point is that there are predictable shifts in preference when the same problem is framed in different ways. In particular, they claim that choices involving gains are often risk-averse while choices involving losses are often risk-seeking. The framing of problems, of course, may influence whether a given outcome is viewed as a gain or a loss.

We are especially interested in the effect of context when decision situations are presented in abstract versus more concrete formulations. For instance, consider the following two alternative ways of phrasing the same choice:

### Insurance Formulation

Situation A: You stand a 1 out of 100 chance of losing $1,000.

Situation B: You can buy insurance for $10 to protect you from this loss.

## Gamble Formulation

Situation A: You stand a 1 out of 100 chance of losing $1,000.

Situation B: You will lose $10 with certainty.

According to EU theory both formulations involve a choice between
$[.01U(W_o - 1000) + .99U(W_o)]$ and $U(W_o - 10)$, where $W_o$ represents the current
wealth level. Hershey and Schoemaker [13] found that under the insurance
formulation, 81% of the subjects preferred situation B, compared with 56%
under the gamble formulation. Apparently individuals focus on protective
aspects when the situation is presented in an insurance context so that this
is perceived as a gain. In the gamble formulation, however, people are more
likely to perceive the $10 as a loss.

Similar results had earlier been obtained by Schoemaker and Kunreuther
[27] with respect to the presentation of information on deductibles in insur-
ance policies. It was found that subjects had a stronger preference for low
deductibles when they were presented in an insurance formulation than in a
pure gamble formulation. Individuals appear to view a lower deductible as
valuable protection against a commensurate portion of the potential loss, but
do not look at it this way when the option is presented as a statistical
lottery. Context focuses attention on different aspects of the problem.

A related explanation of the preference for low deductibles is provided
by Thaler [33] who suggests that it is due to regret considerations. Individ-
uals prefer not to have to think about expenditures when they incur an acci-
dent or, for instance, are in a hospital. Selecting the lowest deductible
minimizes regret in confronting this problem. In this context, it is intrig-
uing to speculate why many individuals who take the lowest deductible often
choose not to collect on their policy after suffering a loss. If they are

concerned that their insurance premium will increase if they make a claim,
then it is not clear why the low deductible policy would have been chosen in
the first place. However, if different considerations are attended to at
different phases of the decision process, this behavior becomes understandable.

In an earlier experiment [13], Hershey and Schoemaker established such
shifts in perspective using a between-subject design. In the follwoing
experiment we test the insurance context effect on a within-subject
basis.

## Experiment 5

We conducted a two-question experiment with 217 Wharton undergraduate
students (mostly sophomores) taking an introductory management course. The
choice involved a .001 chance of losing $5,000 versus a sure loss of $5.
Each subject received this question in both an insurance and a pure gamble
format. To avoid carry-over effects, the questions were interspersed with
several others, and counterbalanced. No order effect was observed.

The within-subject results, shown in Table 6, confirm that individuals
tend to be more risk-averse under the insurance formulation than under the
gamble formulation (p < .01, sign test). Of the 217 pairs of within-subject
responses, 22% were more risk-seeking under the gamble formulation than under
the insurance formulation with only 5% shifting preference in the opposite
direction. These findings are consistent with prospect theory's reference
shift prediction [15, p. 287], as well as the evoking process and societal
norm explanations offered in [13].

The central point emerging from all of these results is that people do
not hold preferences free of context. Whereas expected utility theory
focuses on the decision's structure, individuals are quite sensitive to the

TABLE 6

WITHIN-SUBJECT ANALYSIS OF INSURANCE CONTEXT EFFECT

| Insurance Formulation | Gamble Formulation | | | |
| --- | --- | --- | --- | --- |
| | Prefer Sure Alternative | Indifferent | Prefer Risky Alternative | Total |
| Prefer Sure Alternative | 47% | 2% | 13% | 62% |
| Indifferent | 2 | 2 | 7 | 11 |
| Prefer Risky Alternative | 2 | 1 | 24 | 27 |
| Total | 51% | 5% | 44% | 100% |

NOTE: Each entry denotes the percentage of subjects stating the designated preference combinations (N = 217).

decision's context (Abelson [1], Vlek and Stallen [37], and Einhorn and

Hogarth [6]). Indeed, without context, the choice is not likely to be very

meaningful. Additional illustrations of the importance of decision framing

and context are presented in [34].

## DISCUSSION

In this paper we have raised several methodological and empirical

questions regarding the uniqueness of Von Neumann-Morgenstern utility func-

tions. The starting point for this analysis was several recent behavioral

studies on decisions under risk whose implications for normative theory needed

to be examined.

We showed experimentally that considerable indeterminacy exists as to the

nature of people's risk-taking attitudes. The low convergence of risk-taking

measures across tasks and situations [28] lends general support for this

contention. For instance, Neter and Williams [22] found little correspondence

between insurance choices predicted from expected utility calculations (after

having derived utility functions for their subjects) and the actual choices

made by the subjects in the concrete decision situations. In a related vein,

Dyckman and Salomon [5] showed that utility functions obtained using random

device analogues (e.g., colored chips in a box) were rather different (more

risk-averse) from those based on simulations of actual decision situations.

In the context of constructing utility functions, we examined five

related factors (see Fig. 1) that may influence the shape of the resulting

utility curve. Empirical evidence was provided for each bias, both from

existing literature as well as our experiments. Moreover, it is important

to note that the tests we conducted are quite conservative. In experiments

one, three, four and five the focus is on the percent of risk-averse, risk-neutral, and risk-seeking responses between an experimental and control group. Hence, we only measure influences that are strong enough to induce a shift from one risk-category to another. However, weaker effects may occur as well in terms of the degrees of risk-aversion (or seeking), which would never surface when just looking at categorical responses. A much stronger test would thus be to compare risk-premiums parametrically. This, however, would require specific knowledge as to each subjects' utility function. For example, for a given choice a CE risk-premium (expressed in dollars) can only be compared with a PE risk-premium (expressed in probability points) for consistency once a specific utility function is available. Given the conservativeness of the tests we conducted, the results attest yet more convincingly to the pervasiveness of the biases examined.

In a general sense, all five factors studied should be viewed as context effects, since each is counter to EU theory. Although we examined the factors in isolation, they are likely to interact, either strengthening or counter-acting each other. A fuller understanding of their joint workings will be attempted in future work, for instance through factoral designs.

Of course context effects are not new. Well-known examples include Bar-Hillel's conjunctive and disjunctive probability biases [2], Ronen's sequence bias in compound events [25], Slovic and Lichtenstein's response mode effects [29], the so-called preference reversal phenomenon [11, 19, 20], or deductible effects [27].

In this paper, however, we have attempted to relate contextual biases to the specific problem of constructing utility curves and drawing generalizations of people's risk-taking attitudes. Both, we feel, are plagued with serious

indeterminacy problems which require better descriptive decision theories for their resolution.

The context boundedness of utility functions thus is the major point of our paper. As shown in [5, 11, 13, 15, 20, 25, 29, 34] and our experiments, the way in which a problem is formulated, including its script, presentation, and response mode, affects people's preferences in non-normative ways. Such context dependencies raise serious questions as to the construct validity of the NM utility function. The confounding effects of the measurement process and problem context in particular suggest that in practical applications it is well-advised to seek convergent validation of risk-taking measures. For instance, problems might be presented in various forms to check for consistency. The client could then be asked to reconcile any incompatibilities. Our paper has identified five different areas for such coherency checks. One general strategy for minimizing the biases is to express all choices in terms of final wealth positions. The full implications of our findings, however, need to be further examined in future research. At present, our paper mainly serves to demonstrate the need for a more systematic approach to utility encoding.

## Footnotes

1. The remaining two functions were linear.

2. Each method is thus comprised of a two-step procedure, as is common in actual applications. Although the same binary choice is made under each method prior to the indifference judgment, subjects function all along in either a certainty equivalence mode or a probability equivalence one, both because of the initial example and preceding questions.

3. See Hershey and Schoemaker [13] for further detail.

4. If utility functions are indeed predominantly as F-K propose, then systematic asymmetries between gain and loss lotteries would be expected. The evidence for such a reflection hypothesis, however, is rather weak [14].

5. In the Halter and Dean [12] study, however, both options occasionally had negative expected values, which may explain their higher incidence of risk-seeking curves relative to the other mixed lottery studies (see Table 1).

6. These conclusions are consistent as well with Spetzler's [26] results.

7. Since translations are theoretically identical to wealth effects, the extent of observed reversals of preference rule out that behavior could be explained through utility functions defined on final wealth levels. However, the reversals could be consistent with utility functions defined on changes in wealth, assuming an inflection point at the origin.

8. As with experiments 1 and 3, the risk premiums need not necessarily be the same (from an EU perspective) for the two types of wording, which complicates parametric tests as to any biases.

## REFERENCES

[1] Abelson, R. P. Script Processing in Attitude Formation and Decision Making. Cognition and Social Behavior. Edited by J. S. Carroll and J. W. Payne. Hillsdale, N. J.: Lawrence Erlbaum Associates, 1976, 33-45.

[2] Bar-Hillel, M. On the Subjective Probability of Compound Events. Organizational Behavior and Human Performance, 9, 3, June 1973, 396-406.

[3] Barnes, J. D., and Reinmuth, J. E. Comparing Imputed and Actual Utility Functions in a Competitive Bidding Setting. Decision Sciences, 7, 4, October 1976, 801-812.

[4] Bettman, J. An Information Processing Theory of Consumer Choice. Reading, MA: Addison Wesley, 1979.

[5] Dyckman, T. R., and Salomon, R. Empirical Utility Functions and Random Devices: An Experiment. Decision Sciences, 3, 2, April 1972, 1-13.

[6] Einhorn, H. J., and Hogarth, R. M. Behavioral Decision Theory: Processes of Judgment and Choice. Annual Review of Psychology, 32, 1981, 53-88.

[7] Fishburn, P. C. Mean-Risk Analysis with Risk Associated with Below Target Return. American Economic Review, 67, 1977, 116-126.

[8] Fishburn, P. C., and Kochenberger, G. A. Two-piece Von Neumann-Morgenstern Utility Functions. Decision Sciences, 10, 4, October 1979, 503-518.

[9] Grayson, C. J. Decisions Under Uncertainty: Drilling Decisions by Oil and Gas Operators. Cambridge, MA: Graduate School of Business, Harvard University, 1960.

[10] Green, P. E. Risk Attitudes and Chemical Investment Decisions. Chemical Engineering Progress, 59, 1, January 1963, 35-40.

[11] Grether, D. M., and Plott, C. R. Economic Theory of Choice and the Preference Reversal Phenomenon. American Economic Review, 69, 4, September 1979, 623-638.

[12] Halter, A. N., and Dean, G. W. Decisions Under Uncertainty. Cincinnati, Ohio: South-Western Publishing Co., 1971.

[13] Hershey, J. C., and Schoemaker, P. J. H. Risk Taking and Problem Context in the Domain of Losses: An Expected Utility Analysis. The Journal of Risk and Insurance, 47, 1, March 1980, 111-132.

[14] Hershey, J. C., and Schoemaker, P. J. H. Prospect Theory's Reflection Hypothesis: A Critical Examination. Organizational Behavior and Human Performance, 25, 3, June 1980, 395-418.

[15] Kahneman, D., and Tversky, A. Prospect Theory: An Analysis of Decision Under Risk. Econometrica, 47, 2, 1979, 263-291.

[16] Karmarkar, U. S. Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model. Organizational Behavior and Human Performance, 21, 1, February 1978, 61-72.

[17] Kunreuther, H. C. The Economics of Protection Against Low Probability Events. Working Paper 81-3, International Institute for Applied Systems Analysis, Laxemberg, Austria, 1980.

[18] Laughhunn, D. J., Payne, J. W., and Crum, R. Managerial Risk Preferences for Below-Target Returns. Management Science, 26, 12, December 1980, 1238-1249.

[19] Lichtenstein, S., and Slovic, P. Response-induced Reversals of Preference in Gambling: An Extended Replication in Las Vegas. Journal of Experimental Psychology, 101, 1, November 1973, 16-20.

[20] Lindman, H. R. Inconsistent Preferences Among Gambles. Journal of Experimental Psychology, 89, 2, August 1971, 390-397.

[21] Markowitz, H. The Utility of Wealth. Journal of Political Economy, 60, 2, April 1952, 151-158.

[22] Neter, J., and Williams, C. A., Jr. Acceptability of Three Normative Methods in Insurance Decision Making. The Journal of Risk and Insurance, 38, 3, September 1971, 385-408.

[23] Officer, R. R., and Halter, A. N. Utility Analysis in a Practical Setting. American Journal of Agricultural Economics, 50, 2, May 1968, 257-277.

[24] Payne, J. W., Laughhunn, D. J., and Crum, R. Levels of Aspiration and Preference Reversals in Risky Choice. Management Science, 26, 10, 1980, 1039-1060.

[25] Ronen, J. Effects of Some Probability Displays on Choices. Organizational Behavior and Human Performance, 9, 1, February 1973, 1-15.

[26] Schoemaker, P. J. H. Experiments on Decisions Under Risk: The Expected Utility Hypothesis. Boston, MA: Martinus Nijhoff Publishing Co., 1980.

[27] Schoemaker, P. J. H., and Kunreuther, H. An Experimental Study of Insurance Decisions. The Journal of Risk and Insurance, 46, 4, December 1979, 603-618.

[28] Slovic, P. Information Processing, Situation Specificity, and the Generality of Risk-taking Behavior. Journal of Personality and Social Psychology, 22, 1, April 1972, 128-134.

[29] Slovic, P., and Lichtenstein, S. The Relative Importance of Probabilities and Payoffs in Risk Taking. Journal of Experimental Psychology, 78, 3, Part 2, November 1968, 1-18.

[30] Spetzler, C. S. The Development of a Corporate Risk Policy for Capital Investment Decisions. IEEE Transactions on Systems Science and Cybernetics, SSC-4, September 1968, 279-300.

[31] Spetzler, C. S., and Von Holstein, C. A.  Probability Encoding in Decision Analysis.  Management Science, 22, 1975, 340-358.

[32] Swalm, R. D.  Utility Theory—Insights into Risk Taking.  Harvard Business Review, 47, November-December 1966, 123-136.

[33] Thaler, R.  Toward a Positive Theory of Consumer Choice.  Journal of Economic Behavior and Organization, 1, 1980, 39-60.

[34] Tversky, A., and Kahneman, D.  The Framing of Decisions and the Psychology of Choice.  Science, 211, 1981, 453-458.

[35] Tversky, A., and Sattath, S.  Preference Trees.  Psychological Review, 86, 1979, 542-73.

[36] Van Dam, C.  Beslissen in Onzekerheid.  Leiden, The Netherlands:  H. E. Stenfert Kroese, B. V., 1973.

[37] Vlek, C., and Stallen, P. J.  Rational and Personal Aspects of Risk.  Acta Psychologica, 45, 1-3, 1980, 273-300.

[38] Von Neumann, J., and Morgenstern, O.  Theory of Games and Economic Behavior.  2nd ed.  Princeton, N. J.:  Princeton University Press, 1947.

[39] Wehrung, D. A., MacCrimmon, K. R., and Brothers, K. M.  Utility Measures: Comparisons of Domains, Stability, and Equivalence Procedures.  Working Paper 603, University of British Columbia, 1980.

[40] Williams, C. A.  Attitudes Toward Speculative Risks as an Indicator of Attitudes Toward Pure Risks.  Journal of Risk and Insurance, 33, 4, December 1966, 577-586.